國立高雄大學應用數學系 102 學年度

微積分競賽試題

1. (10%) Find the directional derivative of $f(x,y)=(x-1)y^2e^{xy}$ at $(0,1)$ toward the point $(-1,3)$.

2. (10%) Let f be continuous and define F by
$$F(x)=\int_0^{x^2}\left[t\int_1^t f(u)\,du\right]dt.$$
Find (a) $F'(x)$. (b) $F'(1)$. (c) $F''(x)$. (d) $F''(1)$.

3. (10%) Show if f is continuous on $[0,1]$ and $0\le f(x)\le 1$ for all $x\in[0,1]$, then there exists at least one point c in $[0,1]$ at which $f(c)=c$.

4. (21%) For the function $f(x)=\frac{x^2-3}{x^3}$.
(a) Find the domain and the range of $f(x)$;
(b) Find the vertical and horizontal asymptotes;
(c)Find the x–intercept point and the y–intercept;
(d) Find the critical numbers of f and determine the intervals on which f increasing and the interval on which f decreasing;
(e) Determine the intervals on which the graph of f is concave up and the interval on which the graph of f is concave down;
(f) Determine the vertical tangents and the cusps;
(g) Classify the extreme values and find the points of inflection.

5. (10%) Find the improper integral.
$$(a)\ \int_e^{\infty}\frac{1}{x(\ln x)^2}dx;\qquad (b)\ \int_1^{\infty}\frac{1}{(x-1)^2}dx.$$

6. (10%) Find the limit (if it exists).
$$(a)\ \lim_{x\to 0}\frac{\sqrt{a+x}-\sqrt{a-x}}{x};\qquad (b)\ \lim_{n\to\infty}\left(n^2+n\right)^{1/n}.$$

7. (10%) Find equations for the tangent and normal lines of the equation $\tan xy=x$ at the point $\left(1,\frac{\pi}{4}\right)$.

8. (10%) Find the length of the curve $\overrightarrow{r}(t)=a\cos t\,\overrightarrow{i}+a\sin t\,\overrightarrow{j}+bt\,\overrightarrow{k}$ from $t=0$ to $t=2\pi$.

9. (9%) Determine whether the statement is true or false. If it is false, explain why or given an example that shows it is false:
(a) If $\lim_{x\to c}f(x)=L$, then $f(c)=L$.
(c) If $f(x)=g(x)$ for $x\neq c$ and $f(c)\neq g(c)$, then either f or g is not continuous at c.
(d) If $f'(x)=g'(x)$, then $f(x)=g(x)$.