國立高雄大學應用數學系 103 學年度
微積分競賽試題

學號：________________ 姓名：________________ 編號：____________

題號	1	2	3	4	5	6	7	8	總分
分數									

Announcement: Please write every detail down to receive full credit. If you use any result which was not covered in our lectures so far, you need to give a detailed proof; otherwise you will lose points. Feel free to ask anything about the questions in this paper.

$\boxed{30}$ 1. **True or False.** Determine whether the following statement is true or not. **Sketch your reasoning** if it is true **or give a counterexample** if it is false.

(a) If f is bounded (may not be continuous) on a closed interval $[a,b]$, then f is integrable on $[a,b]$.

(b) If f is continuous and g is integrable on $[a,b]$, then there exists $c \in (a,b)$ such that $\int_a^b f(x)g(x)dx = f(c)\int_a^b g(x)dx$.

(c) Suppose $f(x) = x^x$ is defined for $x > 0$. Then $f'(x) = x^x(1+\ln x)$.

(d) $\int_{-1}^{1} \frac{1}{x}dx = 0$.

(e) Suppose f, g are continuous. If there exists $T \in \mathbb{R}$ such that $\int_a^{a+kT} f(x)dx = \int_a^{a+kT} g(x)dx$ for all $a \in \mathbb{R}, k \in \mathbb{N}$, then $f(x) = g(x)$ for all x.

$\boxed{10}$ 2. Find $\int \frac{3x+2}{x^2+2x+3}dx$.

$\boxed{10}$ 3. Let $n \in \mathbb{N}$ and let $f(x) = x(x-1)(x-2)\cdots(x-n)$. Show that $f'(n) = n!$, where $n! = 1 \cdot 2 \cdot 3 \cdots n$.

$\boxed{10}$ 4. Find $f'(1)$, where $f(x) = \begin{cases} x^2, & x \le 1; \\ 2x-1, & x > 1. \end{cases}$

$\boxed{10}$ 5. Suppose f is differentiable and $\int_1^x e^{\cos t} f(t)dt = x^2 \sin x + \cos^2 x - 2$ for $x \ge 1$. Find $f'(\pi/2)$.

$\boxed{10}$ 6. Determine whether $\Sigma \frac{(-1)^n (\ln n)^3}{n}$ is absolutely convergent, conditionally convergent, or divergent.

$\boxed{10}$ 7. Find two positive numbers such that the sum of the first and twice the second is 36 and the product is a maximum.

$\boxed{10}$ 8. Find the interval of convergence of the series $\sum\limits_{n\ge1} \frac{(x+2)^n}{n4^n}$.