國立高雄大學 101 學年度第 1 學期應用數學系博士班資格考試試題

科目：組合數學　　日期: 101 年 9 月 28 日　　考試時間：180 分鐘

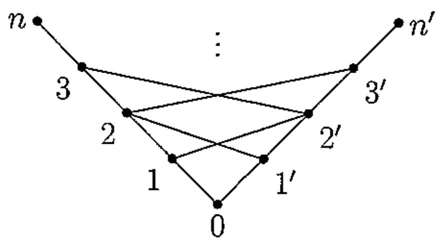
You need to show all your work.

1. (10pts) Show that the number of partitions of n with all parts ≥ 2 equals $p(n) - p(n-1)$.

2. (10pts) Let A_n be the number of domino tilings of a $2 \times n$-rectangle. For example, $A_1 = 1, A_2 = 2, A_3 = 3$. Compute A_n.

3. (10pts) Determine $\sum_{n\geq 0} \binom{2n+1}{n} z^n$.

4. (10pts) Compute the Möbius function for the poset



that is, $i < j$, $i < j'$, $i' < j$, $i' < j'$, if $i < j$ as natural numbers.

5. (15pts) Let $s_{n,k}$ be the signless Stirling numbers of the first kind. Use the polynomial method to show that $s_{n+1,k+1} = \sum_{i=0}^{n} \binom{i}{k} s_{n,i}$. Can you find a combinatorial proof?

6. (15pts) Let $S_{n,k}$ be the Stirling numbers of the second kind. Verify $S_{n,k} = \sum 1^{a_1-1} 2^{a_2-1} \cdots k^{a_k-1}$, where the sum extends over all solutions of $a_1 + a_2 + \cdots + a_k = n$ in positive integers.

7. (15pts) Let $h_k(n)$ be the number of permutations in $S(n)$ all of whose cycle lengths are divisible by k. Compute $h_k(n)$.

8. (15pts) How many 2-colored face patterns of the cube exist when white and black appear equally often? The same question with edges and vertices.