國立高雄大學 103 學年度第 2 學期應用數學系博士班資格考試試題

科目：組合數學　　　　　　日期: 104 年 3 月 4 日　　　　考試時間：180 分鐘

You need to show all your work.

1. (10 pts) Express the following quantity in terms of Fibonacci number:
   The number of ordered partitions of n into parts greater than 1.

2. (15 pts) Let $S_{n,k}$ be the Stirling numbers of the second kind.

   (a) Find a formula for $S_{n,3}$.

   (a) Find a closed formula for $S_{n,n-2}$.

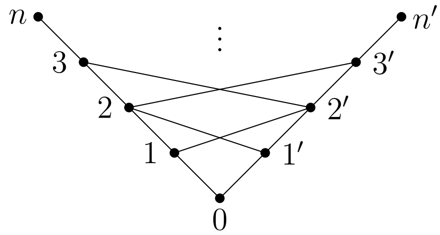
3. (10 pts) Find the unique sequence (a_n) of real numbers with
$$\sum_{k=0}^{n} a_k a_{n-k} = 1 \quad (n \geq 0).$$

4. (15 pts) Determine the numbers $a_n \in \mathbb{N}_0$ from the identity
$$n! = a_0 + a_1 n^{\underline{1}} + a_2 n^{\underline{2}} + \cdots + a_n n^{\underline{n}} \quad (n \geq 0).$$

5. (10 pts) Prove the formula $\mathrm{Bell}(n) = \dfrac{1}{e} \displaystyle\sum_{k \geq 0} \frac{k^n}{k!}$.

6. (15 pts) Prove the following identities:
$$\prod_{i \geq 1} \frac{1}{1 - qz^i} = \sum_{k \geq 0} \frac{q^k z^k}{(1-z)\cdots(1-z^k)} = \sum_{k \geq 0} \frac{q^k z^{k^2}}{(1-z)\cdots(1-z^k)(1-qz)\cdots(1-qz^k)}.$$

7. (10pts) Compute the Möbius function for the poset



   that is, $i < j$, $i < j'$, $i' < j$, $i' < j'$, if $i < j$ as natural numbers.

8. (15 pts) Let G be the symmetry group of the cube. Compute the cycle index when N is the set of edges.