一百一十二學年度第一學期微積分會考試題

一、單選擇題 (單選十題，每題五分，共五十分，答錯不倒扣)

1. Let $f(x)=\int_0^{x^2}\frac{\sin(t)}{(t+1)^2}dt$. Evaluate $\lim_{x\to\infty}f'(x)=$

   (A) 2; (B) 1; (C) 0; (D) None of the above.

2. Evaluate $\lim_{x\to 0}\frac{\csc(x)-\cot(x)}{x}$

   (A) 0; (B) 1; (C) ∞; (D) None of the above.

3. Let $f(x)=\frac{1}{4}x^3+x-1$. Find $(f^{-1})'(3)$.

   (A) 1; (B) $\frac{1}{2}$; (C) $\frac{1}{4}$; (D) None of the above.

4. The integral $\int_1^9\frac{1}{\sqrt{x}(1+\sqrt{x})^2}\,dx$ =

   (A) 2; (B) $\frac{1}{2}$; (C) $\frac{1}{4}$; (D) None of the above.

5. Consider $f(x)=\frac{1-\cos(x)}{x^2}$ with $x\neq 0$. Which of the following statements is **true** ?

   (A) $f(x)\in[0,\frac{1}{2})$;

   (B) $f(x)$ is a inflection point at $x=1$;

   (C) $f(x)$ has exactly one root;

   (D) $f(x)$ has a slant asymptote $y=x$.

6. Evaluate $\lim_{n\to\infty}\frac{1}{n^3}(1^2+2^2+\cdots+n^2)$ =

   (A) $\frac{1}{3}$; (B) $\frac{1}{6}$; (C) 0; (D) ∞.

7. Let $G(x) = \int_0^x \left[s \int_0^s \frac{1}{t^2+t+2} dt \right] ds$, find $G''(0) =$

(A) -1; (B) 1; (C) $\sqrt{2}$; (D) None of the above.

8. On what interval is the curve $f(x) = 2x^{5/3} - 5x^{4/3}$ concave upward?

(A) $(0,1)$; (B) $(1, \infty)$; (C) $(-\infty, 0)$; (D) $(-\infty, -1)$.

9. Find the equation of the tangent line to the curve $y = e^{(-2x+x^2)}$ at $x = 2$.

(A) $y = -x + 3$; (B) $y = 3x - 5$; (C) $y = x - 1$; (D) $y = 2x - 3$.

10. Find the slope of the graph of $3(x^2 + y^2)^2 = 100xy$ at the point $(3,1)$.

(A) $\frac{13}{9}$; (B) $\frac{61}{45}$; (C) $\frac{-61}{45}$; (D) $\frac{2}{9}$.

二、多選擇題 (多選五題，每題六分，共三十分。答錯一個選項扣三分，錯兩個選項以上不給分，分數不倒扣)

11. Which of the following statements are **not true** ?

(A) If $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = L$, then f is continuous at $x = a$;

(B) If a function is continuous on a closed interval, then it must have a minimum on the interval;

(C) If *y* is a differentiable function of *u*, and *u* is a differentiable function of *x*, then *y* is a differentiable function of *x*;

(D) If f has an inverse function, then $(f^{-1})^{-1} = f$ and $f^{-1} = \frac{1}{f}$.

12. Which of the following statements are **true** ?

(A) If $f(1) = 1$ and $3 \geq f'(x)$ for $all\ x$, then $f(5) \leq 13$;

(B) If f is continuous on $[0,1]$, then $\int_0^1 [f(x) + f(1-x)]dx = 0$;

(C) If $f(a) < k < f(b)$ with $a < b$, then there is at least one number $c\ in\ [a,b]$ such that $f(c) = k$;

(D) $\frac{d}{dx}\left[\int_{u(x)}^{v(x)} f(t)dt\right] = f(x)v'(x) - f(x)u'(x)$.

13. Which of the following statements are **not true** ?

(A) If f is differentiable at a, so is $|f|^2$;

(B) If $x = a$ is a critical point of $y = f(x)$, then f has a minimum at $x = a$;

(C) If f is differentiable at a, then f is continuous at a;

(D) If f is differentiable at a, then $f'(a) = \lim_{x \to a} f'(x)$;

14. Suppose that $f'(a) = 1$ for some constant a. The **possible values** for the limit $\lim_{x \to a} \frac{f(x)-f(a)}{e^x-1}$ are

(A) 1; (B) e; (C) e^{-1}; (D) 0.

15. Consider

$$f(x) = \begin{cases} 1, & if\ x = 0; \\ \frac{\sin(x)}{x}, & if\ x \neq 0. \end{cases}$$

Which of the following statements are **true**?

(A) $f(x)$ is differentiable at $x = 0$;

(B) $f(x)$ is continuous at $x = 0$;

(C) $f(x)$ has a global maximum value;

(D) $f(x)$ has an inflection point.

三、填空題 (五題，每題四分，共二十分，答錯不倒扣)

1. For what value of c does the polynomial $P(x) = x^4 + cx^3 + x^2$ have two inflection points? ___(1)____.
2. Solve the equation $\ln(x) + \ln(x-1) = 1$. $x =$___(2)____.
3. The integral $\int_1^3 \frac{1}{\sqrt{t}(1+t)} dt =$___(3)____.
4. Let $F(x) = \int_2^x \sqrt{1+t^2}\mathrm{dt}$. Then the derivative $(F^{-1})'(0)=$ ___(4)____.
5. The integral $\int \frac{x}{\sqrt{9+8x^2-x^4}} dx =$ ___(5)____.