

國立高雄大學應用數學系

有獎徵答

(2014 年 12 月)

1. 將於學期間公告6道題目，由系上老師出題，歡迎同學來挑戰。
2. 同學解出任何一題皆可將解答投稿到系辦 (答案紙格式可於系網頁上下載)。或利用電子郵件 math@nuk.edu.tw (主旨：○○○年○月第○題解答)

01

Consider the 2nd order ODE

$$2z \frac{d^2 y}{dz^2} + \frac{dy}{dz} - n^2 y = 0, \quad z > 0.$$

By the change of independent variable of the form $z = a x^m$ with suitable choice of constants a and m , show that the general solution of the equation can be written as

$$y = A e^{n \sqrt{2z}} + B e^{-n \sqrt{2z}},$$

where A and B are constants to be determined with initial conditions.

[陳晴玉教授出題，微分方程]

02

Prove that, when $p > 0$, the improper integrals

$$\int_1^\infty \frac{\sin x}{x^p} dx \quad \text{and} \quad \int_e^\infty \frac{\cos x}{\ln^p x} dx$$

are both convergent.

[施信宏教授出題，高等微積分]