Pseudospectral matrix element method coupled with direct-forcing immersed boundary method for a long flexible cylinder

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Abstract

Vortex-induced vibration(VIV) of flexible cylinder is a very complexproblem in fluid mechanics, the present study result of flow-induced vibration of a long flexible cylinder at different Reynolds numbers. This study aims to use the pseudopsectral methods coupled with the direct-forcing immersed boundary (DFIB) method to investigate this phenomenon. We simulated Reynolds number = 100 for 500 time units, and increased Reynolds number to 200.

Governing equation and immersed boundary method

1. Governing equation

Navier–Stokes equation and continuity equation can be transformed as follows

$$\partial \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u}\mathbf{u}) = -\frac{1}{\rho}\nabla p + \frac{1}{\mathrm{Re}}\nabla^2 \mathbf{u} + f$$

2. Direct-forcing immersed boundary method

The virtual forcing term, **f**, is determined by the difference between the velocity on the solid boundary point and the desired boundary velocity

$$\mathbf{f} = \eta \frac{\mathrm{U}_s^{n+1} - \mathrm{U}^{**}}{\Delta t}$$

The integral of the virtual force, **F** is the hydrodynamic resultant force

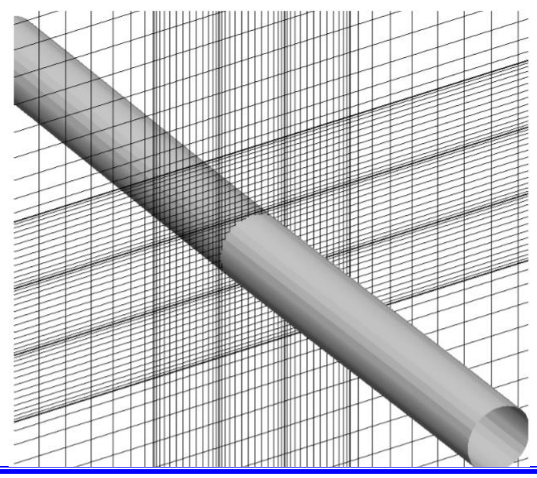
$$\mathbf{F} = \iiint_{\Omega_s} \mathbf{f} \cdot \mathrm{dV}$$

3. Equations of motion for a flexible cylinder

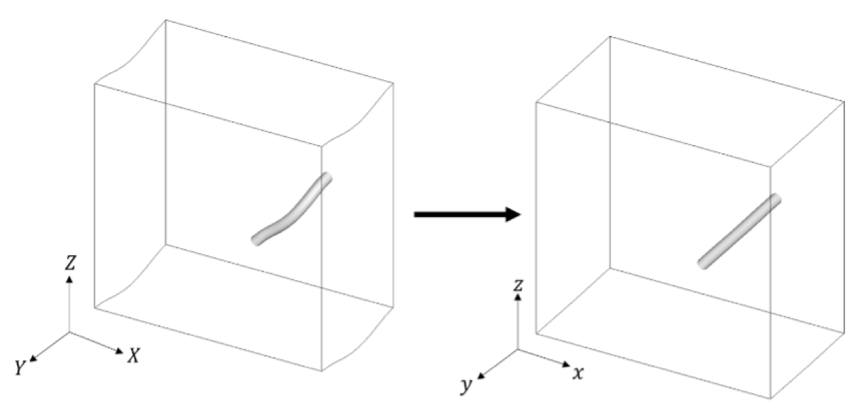
The motion of flexible cylinder is governed by following dimension equation .

$$\ddot{\zeta}(y,t) - c^2 \frac{\partial^2 \zeta(y,t)}{\partial y^2} + b^2 \frac{\partial^4 \zeta(y,t)}{\partial y^4} + \frac{\kappa D}{m_s U_\infty} \dot{\zeta}(y,t) = \frac{2 C_l(y,t)}{\pi m_r}$$

Domain decomposition



Coordinate transformation



Mathematical model

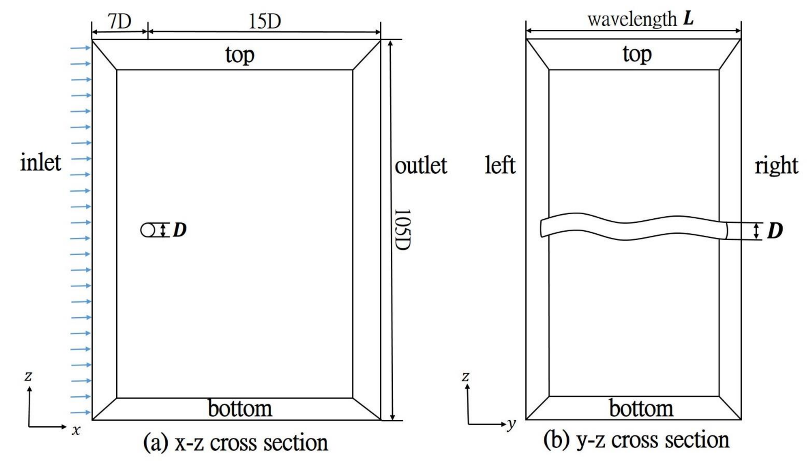


Table 1: Boundary conditions for flow through a cylinder.

	Velocity boundary	Pressure boundary
Top	$(U_\infty, 0, 0)$	$\partial p/\partial n = 0$
Bottom	$(U_\infty, 0, 0)$	$\partial p/\partial n = 0$
Inlet	$(U_\infty, 0, 0)$	$\partial p/\partial n = 0$
Outlet	$\partial \mathbf{u}/\partial n = 0$	$\partial p/\partial n = 0$
Left	periodic	periodic
Right	periodic	periodic

Table 2: Computational domain for flow through a cylinder.

Upstream	7.0D
Downstream	15.0D
Crossflow	105.0D
Spanwise	wavelength $L = 4\pi$

Pseudospectral matrix element method

$\mathbf{h}(x)$ is considered in the domain $\mathbf{h} \in [-1,1]$.

$$h_j = h(x_j).$$

The collocation points x_j of Chebyshev polynominals can be represented as

$$x_j = \cos\left(\frac{j\pi}{N}\right), \quad j=0,1,2,\ldots,N.$$

Chebyshev polynomials T_{jk} can be represented as

$$T_{jk} = \cos\left(\frac{j\pi k}{N}\right), \quad k, j=0,1,2,\ldots,N.$$

$\mathbf{h}(x)$ can approximated by using superposition of Chebyshev polynomials and dented as

$$\mathbf{h} = \mathbf{T}\hat{\boldsymbol{h}}$$

Where $\mathbf{T}$ is the matrix formed by Chebyshev polynomials and $\hat{\boldsymbol{h}}$ is a vector of Chebyshev coefficients.

In matrix notation, $\mathbf{h}$ can be represented as

$$\hat{\mathbf{h}} = \hat{\mathbf{T}}\mathbf{h}$$

$$\widehat{h^{(q)}} = G^{(q)}\hat{\mathbf{h}}$$

$$\frac{d^{(q)}\mathbf{h}}{dx^{(q)}} = \mathbf{T}\widehat{h^{(q)}} = TG^{(q)}\hat{\mathbf{h}} = TG^{(q)}\hat{T}h = \widehat{G^{(q)}}h$$

$$\widehat{G^{(q)}}h = TG^{(q)}\hat{T}$$

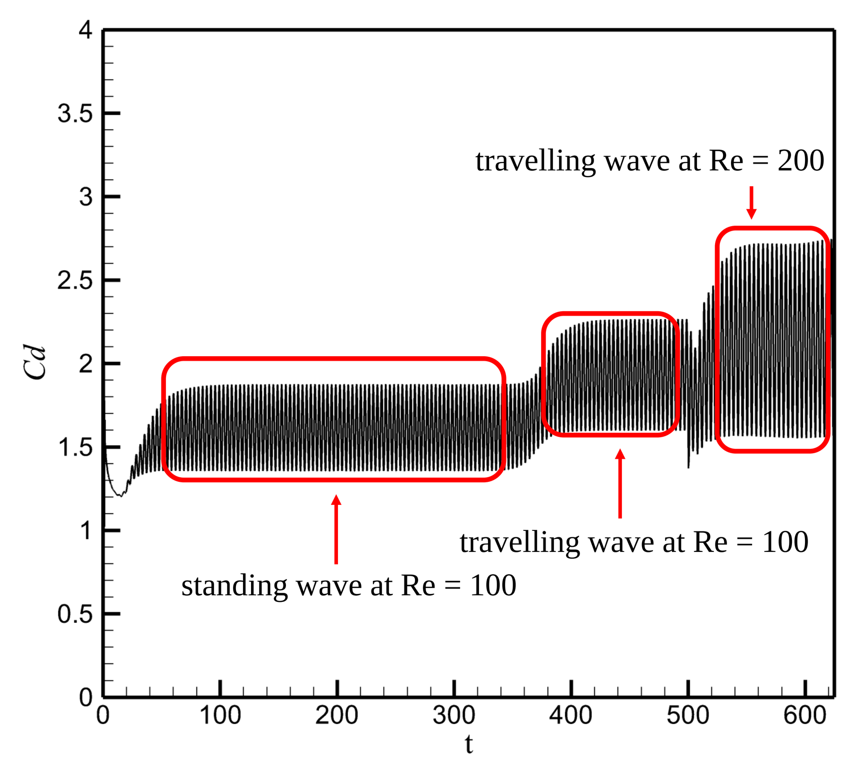
$$G^{(q)} = (G^{(1)})^q$$

$$\frac{d^{(2)}\mathbf{h}}{dx^{(2)}} = \mathbf{T}\widehat{h^{(1)}} = TG^{(2)}\hat{\mathbf{h}} = TG^{(2)}\hat{T}h = \widehat{G^{(2)}}h$$

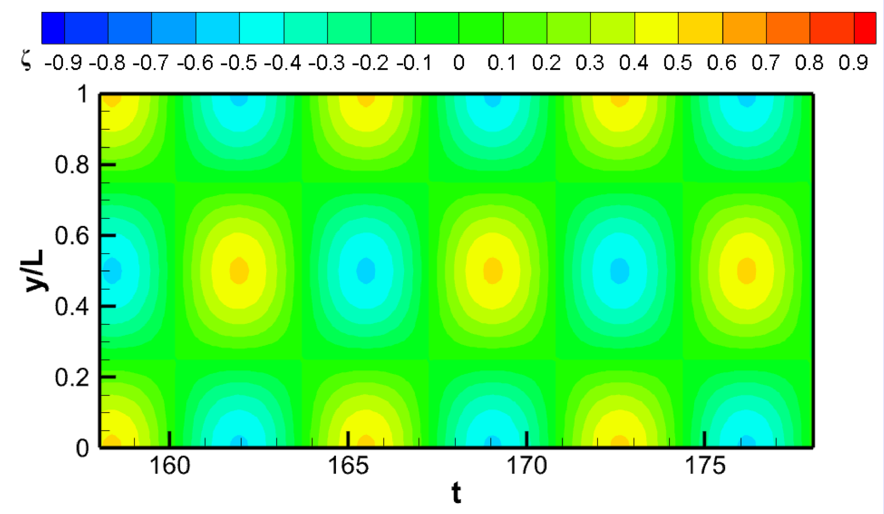
$$\widehat{G^{(2)}} = TG^{(1)}G^{(1)}\hat{T}$$

Results and discussion

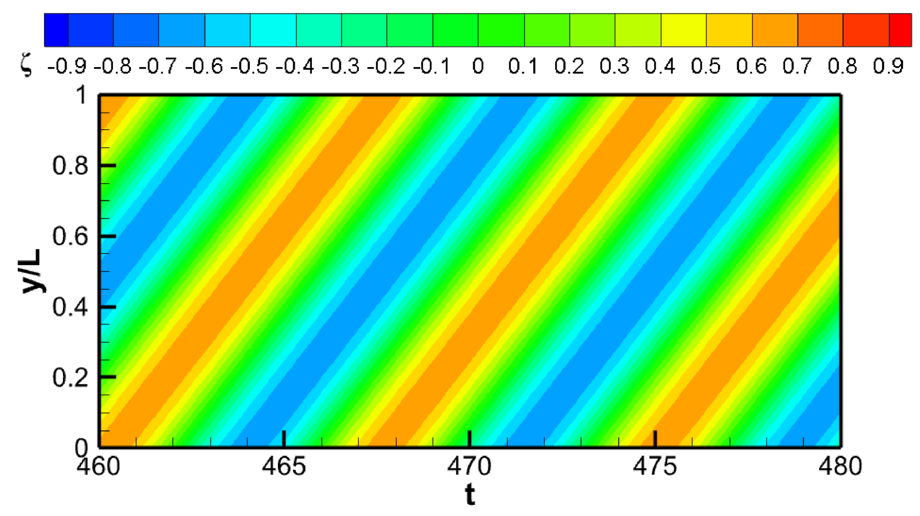
Comparison of displacement, drag coefficient, lift coefficient at Re =100 and 200 at mass ratio = 2.0 in 3D flexible cylinder.



Quantity	Re = 100	Re = 200
std y/d	0.47	0.7
max y/d	0.67	1.00
std C_l	0.7	1.28
max C_l	1.01	2.01
$\bar{C}_d$	1.93	2.13
std C_d	0.23	0.4
max C_d	2.27	2.75
Total grids	819,315	819,315



standing wave



travelling wave

Conclusions

1. This present study combined PSME-DFIB scheme and apply it to solve fluid-structure interactions involving long flexible cylinder at different Reynolds numbers.We obtained drag coefficient, lift coefficient and maximum displacement at Re = 100 and 200. Compared the maximum displacement with the experimental results and get similar results (see Newman et al.[1]).
2. We present a simulations for vortex-induced vibration of a flexible cylinder and consider the initial displacement in standing wave at Re = 100, after a few cycle standing wave transformed into traveling wave, this phenomenon is consistent with the experimental results.
3. Using coordinate transformation method applied to increase the accuracy of the results.
4. The domain decomposition method can greatly save the number of grids.
5. The initial conditions of the traveling wave must be given at Re = 200 otherwise, the result can not converge.

References

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