Simulate Ulam Spiral with GeoGebra and Python

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Abstract

The Ulam spiral is some graphic patterns reflecting the distribution of prime numbers, firstly investigated by mathematician Stanisław Ulam in 1963. The most interesting point is that it connects the number theory and the computer simulation. Because to show the graphic patterns of Ulam spiral or some similar prime spirals depend heavily on the efficiency of computer calculating.

In this work we simulate some prime spirals with the aid of GeoGebra and Python. It is amazing to let students know that they can make some famous mathematical experiment with shareware freely and easily on their desktop computer.

Keywords: Ulam Spiral; Prime spirals; Simulation; GeoGebra; Python

Problem description

The Ulam spiral was firstly drawing by Ulam by writing the positive integers in a spiral arrangement on a square lattice. In figure 1 we show a hand writing pattern picture by chapturing picture From YouTube channel "Numberphile" [1]. Figure 2 is an example in [2]. By computer, the 150 by 150 size of Ulam spiral shows some vivid pattern as in Fig 3 cited from Wiki [2].

Robert Sacks in 1994 devised an Archimedean prime spiral by writing numbers along Archimedean spiral to get another pattern. In Sacks spiral, every full round occures at a perfect square number. The pattern of Sacks spiral are shown in Fig. 4 cited from [2].

We try to simulate some prime spirals with the aid of GeoGebra [3] and Python [4]. In this way, we can check if we can easily repeat the same computer by Ulam and Myron Stein and Mark Wells in 1963 used MANIAC II at Los Alamos Scientific Laboratory [2], on our desktop computer with free shareware in 2021.

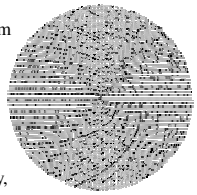


Figure 4: the Sack spiral in [2]

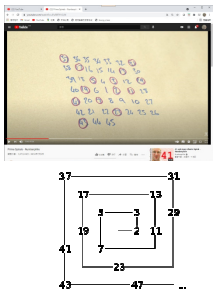


Figure 1, 2: Hand writing pattern In [1] and prime pattern in [2].

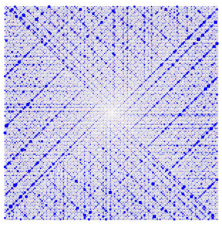
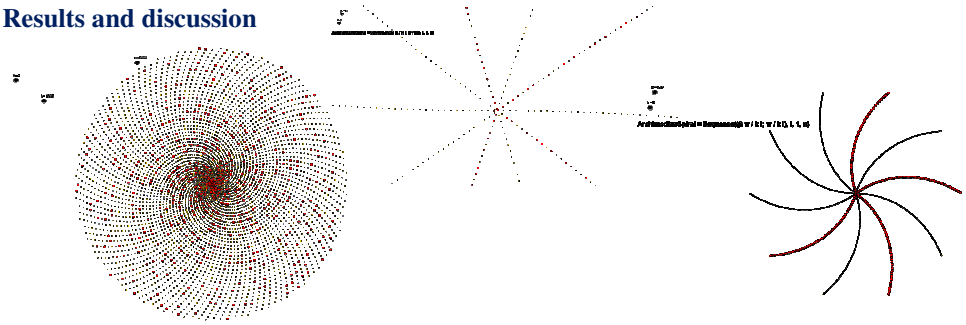


Figure 3: the 150 by 150 size of Ulam spiral in [2].

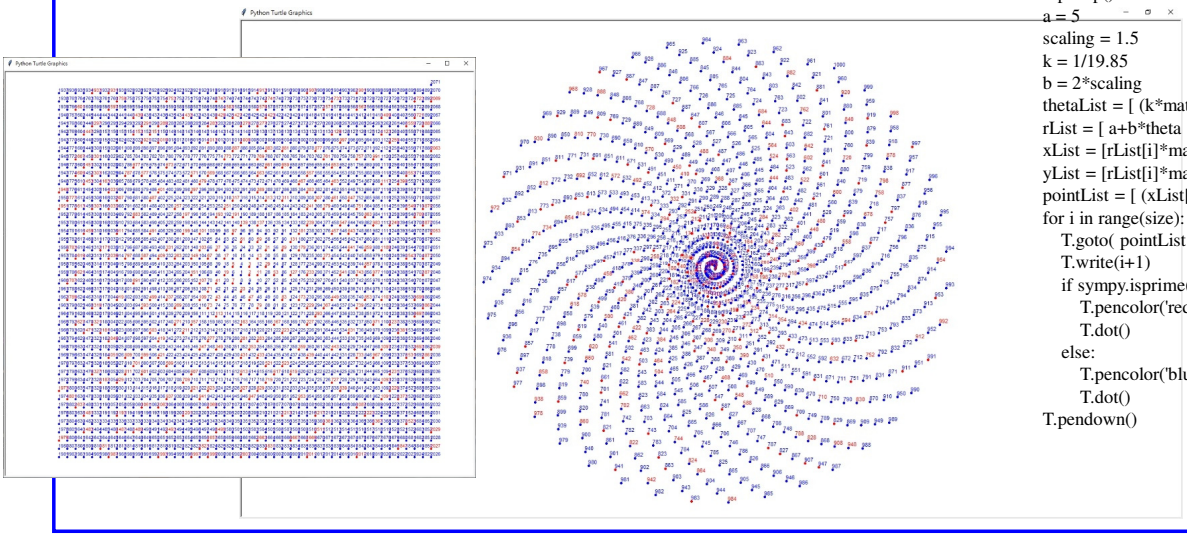
Results and discussion



The figures at the right are generated by shareware GeoGebra. GeoGebra can easily show some clear patterns of Archimedean prime spirals with the "Sequence" instruction:
ArchimedeanSpiral = Sequence((β π / k i; π / k i), i, 1, n)\\
r = β (π / k) i,\; i = 1, \cdots, n\\
\theta = (π / k) i, \;i = 1, \cdots, n\\

The figures below are generated by Python with codes:

```
size = 1000
turtle.tracer(0,0)
T = turtle.Turtle()
T.speed(0)
T.shape('turtle')
T.color('blue', 'green')
T.penup()
a = 5
scaling = 1.5
k = 1/19.85
b = 2*scaling
thetaList = [ (k*math.pi)*i for i in range(size)]
rList = [ a+b*theta for theta in thetaList]
xList = [rList[i]*math.cos(thetaList[i]) for i in range(size)]
yList = [rList[i]*math.sin(thetaList[i]) for i in range(size)]
pointList = [ (xList[i], yList[i]) for i in range(size) ]
for i in range(size):
    T.goto( pointList[i] )
    T.write(i+1)
    if sympy.isprime(i+1):
        T.pencolor('red')
        T.dot()
    else:
        T.pencolor('blue')
        T.dot()
T.pendown()
```



Conclusions

We simulate some prime spirals with the aid of GeoGebra and Python. We can find some pattern with the aid of shareware GeoGebra and Python easily. This computer experiment can be an interesting topic added in Number theory or Scientific Computing course.

References

1. Prime Spirals – Numberphile, https://youtu.be/iFuR97YcSLM.
2. Ulam spiral, https://en.wikipedia.org/wiki/Ulam_spiral.
3. https://www.geogebra.org/
4. https://www.python.org/