

國立高雄大學一〇四學年度第二學期  
理學院微積分基礎能力會考試題

系級：

學號：

姓名：

**I Fundamentality (60%)**

- Evaluate the integral  $\int_0^4 \frac{x}{x+9} dx$ .  
A.  $4 - 13 \ln 9 + 3 \ln 18$    **B.  $4 - 9 \ln 13 + 18 \ln 3$**    C.  $13 - 9 \ln 4 + \ln 3$    D.  $13 - 4 \ln 9 + \ln 3$
- Evaluate the integral  $\int_0^{\pi/4} \tan x \sec^2 x dx$ .  
A. 0   B. 1   **C.  $1/2$**    D.  $1/3$
- Find the volume of the solid generated by revolving the region bounded by the graphs of  $y = \cos x^2$ ,  $x \geq 0$ , and  $y = 0$  about the  $y$ -axis.  
A.  $1/2$    B. 2   **C.  $\pi$**    D.  $\pi/2$
- Find the area of the surface obtained by rotating the curve  $y = e^x$  about the  $y$ -axis for  $1 \leq y \leq 3$ .  
A.  $\int_1^3 2\pi e^x \sqrt{1+e^{2x}} dx$    B.  $\int_0^{\ln 3} 2\pi e^x \sqrt{1+e^{2x}} dx$    C.  $\int_1^3 2\pi x \sqrt{1+e^{2x}} dx$    **D.  $\int_0^{\ln 3} 2\pi x \sqrt{1+e^{2x}} dx$**
- Evaluate the improper integral  $\int_1^\infty \frac{dx}{x \ln x}$ .  
A.  $1/2$    B. 0   C.  $1/4$    **D. divergent**
- Find the length of the curve  $y = \int_1^x \sqrt{t^3 - 1} dt$  for the interval  $1 \leq x \leq 4$ .  
A.  $62/13$    B.  $13/62$    **C.  $62/5$**    D.  $5/62$
- Evaluate the integral  $\int_0^4 (x^2 + 1)e^{-x} dx$ .  
A.  $-27e^{-4} - 1$    **B.  $-27e^{-4} + 3$**    C.  $27e^{-4} + 3$    D.  $-27e^4 + 3$
- Which of the following is an improper integral?  
**A.  $\int_0^4 \frac{1}{x-2} dx$**    B.  $\int_4^8 \frac{\sqrt{x-4}}{x} dx$    C.  $\int_0^{1/2} \sin^{-1} x dx$    D.  $\int_0^4 \tan^7 x dx$
- Evaluate the improper integral in Problem 8.  
A.  $4 - \pi$    B.  $\frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$    C. 0   **D. divergent**
- Evaluate the integral  $\int_{-\pi/2}^{\pi/2} \sin^5 x \cos^2 x dx$ .  
**A. 0**   B.  $1/6$    C.  $1/3$    D. 1
- Find the area of the surface obtained by rotating the curve  $y = \frac{1}{4}x^2 - \frac{1}{2} \ln x$  about the  $y$ -axis for  $1 \leq x \leq 2$ .  
A.  $\frac{10}{3}$    B.  $\frac{11\pi}{2}$    C.  $\frac{3\pi}{2}$    **D. none of the above**

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12. Find  $\lim_{x \rightarrow \pi} \frac{\sin x}{1 - \cos x}$ .  
A. 1    **B. 0**    C.  $\infty$     D. inconclusive
13. Evaluate the integral  $\int_0^{\pi/3} \tan^2 2x \, dx$ .  
A.  $\sqrt{3} - \frac{\pi}{3}$     B.  $\frac{\sqrt{3}}{2} - \frac{\pi}{3}$     C.  $-\frac{\sqrt{3}}{2} - \frac{\pi}{3}$     **D. divergent**

**Problems 14-15:** Car A and Car B move along a straight track, accelerating from rest at a common straight line with velocity  $v_A(t) = 3t$  ft/s and  $v_B(t) = \frac{t^2}{20}$  ft/s, respectively.

14. Which car is ahead of the other after 60 seconds?  
**A. Car A**    B. Car B    C. same position    D. inconclusive
15. Find the time  $T$  that Car A and Car B meet at.  
**A. 90**    B. 105    C. 75    D. 60

## II Medium (24%)

16. Find the length of the curve  $x = \frac{y^4}{8} + \frac{1}{4y^2}$  over  $1 \leq y \leq 2$ .  
A. 1/2    **B. 33/16**    C. 8/3    D. 2
17. Find  $a > 0$  such that  $\int_0^{\infty} \frac{dx}{x^2 + a^2} = 1$ .  
**A.  $\pi/2$**     B.  $\pi/3$     C.  $\pi/4$     D.  $\pi/6$
18. Evaluate the integral  $\int_0^{1/2} \tan^{-1} 2x \, dx$ .  
A.  $\frac{1}{8}(\pi - \ln 2)$     B.  $\frac{1}{8}(\pi + \ln 2)$     **C.  $\frac{1}{8}(\pi - \ln 4)$**     D. divergent
19. Find the integral  $\int \frac{3x - 1}{x^2 + 2x} \, dx$ .  
A.  $\frac{7}{2} \ln|x + 2| + C$     B.  $\frac{1}{2}(7 \ln|x + 2| + \ln|x|) + C$     **C.  $\frac{1}{2}(7 \ln|x + 2| - \ln|x|) + C$**   
D. none of the above
20. Evaluate the integral  $\int_0^{\pi/2} \frac{\cos \theta}{\sin^2 \theta - 6 \sin \theta + 8} \, d\theta$ .  
A.  $\frac{1}{2} \ln 2$     B.  $\frac{1}{2} \ln 3$     **C.  $\frac{1}{2} \ln \frac{3}{2}$**     D.  $\frac{1}{2} \ln \frac{2}{3}$
21. Evaluate  $\lim_{x \rightarrow \infty} (\ln(3 + \sin x))^{1/x}$ .  
A. 0    **B. 1**    C. divergent    D. none of the above

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III Challenge (16%)

22. Find the area of the region that is enclosed between the  $x$ -axis and the curve  $y = \frac{\ln x - 1}{x^2}$  for  $x \geq 1$ .  
A.  $1/e$    **B.  $2/e$**    C.  $e$    D.  $0$
23. Evaluate the integral  $\int_{-\infty}^{\infty} \frac{dx}{4x^2 + 4x + 5}$ .  
A.  $\pi/2$    **B.  $\pi/4$**    C.  $\pi/8$    D. divergent
24. If the infinite curve  $y = e^{-x}$ ,  $x \geq 0$ , is rotated about the  $x$ -axis, find the area of the resulting surface.  
**A.  $\pi(\sqrt{2} + \ln(1 + \sqrt{2}))$**    B.  $\frac{\pi}{6}(\sqrt{2} + \ln(1 + \sqrt{2}))$    C.  $\frac{\pi}{4}(\sqrt{2} + \ln(1 + \sqrt{2}))$    D.  $\frac{\pi}{3}(\sqrt{2} + \ln(1 + \sqrt{2}))$
25. Evaluate the integral  $\int_{\pi/6}^{\pi/3} \frac{\ln \tan x}{\sin x \cos x} dx$ .  
**A.  $0$**    B.  $-\frac{1}{8}(\ln 3)^2$    C.  $\frac{1}{8}(\ln 3)^2$    D.  $-\frac{1}{6}(\ln 3)^2$