國立高雄大學一〇五學年度第一學期
理學院微積分基礎能力會考試題 (A)

日期：105 年 12 月 7 日　　時間：120 分鐘　　學號：　　姓名：

Notation:

- $\mathbb{Z}$: the set of integers (整數集)
- $\mathbb{Q}$: the set of rational numbers (有理數集)
- $\mathbb{R}$: the set of real numbers (實數集)
- $\in$: belong (屬於); for example, $a \in \mathbb{R}$ means that "a is a real number."

I Basic (45%)

1. What is the limit of $f(x) = 2x + 1$ as x approaches 0?

   A. $\lim_{x\to 0}(2x+1) = 0$　**B.** $\lim_{x\to 0}(2x+1) = 1$　C. $\lim_{x\to 0}(2x+1) = 2$　D. none of the above

2. What is the limit of $f(x) = \dfrac{x}{|x|}$ as x approaches 0 from the left?

   A. $\lim_{x\to 0^-} \dfrac{x}{|x|} = -1$　B. $\lim_{x\to 0^-} \dfrac{x}{|x|} = 0$　C. $\lim_{x\to 0^-} \dfrac{x}{|x|} = 1$　D. $\lim_{x\to 0^-} \dfrac{x}{|x|}$ does not exist

3. Which one of the following functions is not continuous on $\mathbb{R}$?

   A. $f_1(x) = \sin x$　**B.** $f_2(x) = \dfrac{1}{x}$　C. $f_3(x) = \dfrac{1}{x^2+1}$　D. none of the above

4. Find the horizontal asymptotes (if any) of the function $f(x) = \sqrt{x^6 + 5x^3} - x^3$, $x \geq 0$.

   A. no horizontal asymptote　B. $y = -\dfrac{5}{2}$　**C.** $y = \dfrac{5}{2}$　D. $y = 0$

5. Find the derivative of $f(x) = \sqrt{10}$.

   A. $\sqrt{10}$　B. $\dfrac{1}{\sqrt{10}}$　C. $\dfrac{1}{2\sqrt{10}}$　**D. none of the above**

6. Find an equation for the line that is tangent to the curve $y = x^3 - x$ at the point $(-1, 0)$.

   A. $y = -\dfrac{1}{2}(x+1)$　B. $y = \dfrac{1}{2}(x+1)$　C. $y = -2(x+1)$　**D.** $y = 2(x+1)$

7. Find the derivative of $f(x) = \sin^2(\pi x - 2)$.

   A. $2\sin(\pi x-2)\cos(\pi x-2)$　**B.** $2\pi\sin(\pi x-2)\cos(\pi x-2)$　C. $2\pi\cos(\pi x-2)$　D. $2\pi\sin(\pi x-2)$

8. Determine the slope of the graph of $x^2y^2 - 2x = 4 - 4y$ at the point $(2, -2)$.

   A. $\dfrac{7}{6}$　B. $-\dfrac{7}{6}$　C. $\dfrac{6}{7}$　D. $-\dfrac{6}{7}$

9. Find all the critical numbers (also called critical points) of $f(x) = \sin|x|$.

   A. $x = \dfrac{\pi}{2} + n\pi$, $n \in \mathbb{Z}$　**B.** $x = 0,\ \dfrac{\pi}{2} + n\pi,\ n \in \mathbb{Z}$　C. $x = \pm\left(\dfrac{\pi}{2} + n\pi\right)$, $n \in \mathbb{N} \cup \{0\}$

   D. $x = 0,\ \pm\left(\dfrac{\pi}{2} + n\pi\right)$, $n \in \mathbb{N} \cup \{0\}$

10. Which one of the following functions has no inflection point at $x = 0$?

    A. $f_1(x) = \sin x$　B. $f_2(x) = x^{5/3}$　**C.** $f_3(x) = x^4$　D. none of the above

11. The graph of the function $f(x) = \cos x$ on the interval $(\pi/2, \pi)$ is

    A. increasing, concave upward　B. increasing, concave downward　**C. decreasing, concave upward**　D. decreasing, concave downward

12. Let $y = \sqrt{x}$. Find the differential dy of y at $x = 4$ with $dx = 3$.
A. $\frac{1}{4}$ **B.** $\frac{3}{4}$ C. $\sqrt{7} - 2$ D. $3(\sqrt{7} - 2)$

13. Express the limit $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^4}{n^5}$ as a definite integral.
A. $\int_0^1 x^4\, dx$ B. $\int_0^1 x^5\, dx$ C. $\int_0^1 \frac{1}{x^4}\, dx$ D. $\int_0^1 \frac{1}{x^5}\, dx$

14. Which one of the following statement is false?
A. $\int_\pi^\pi \sin x\, dx = 0$ B. $\int_0^1 \sqrt{1-x^2}\, dx = -\int_1^0 \sqrt{1-x^2}\, dx$ **C.** $\int_{-1}^1 \frac{1}{x^2}\, dx = \left[-\frac{1}{x}\right]_{-1}^1 = -1 - 1 = -2$ D. none of the above

15. Evaluate the definite integral $\int_1^2 x^3\sqrt{x^4+5}\, dx$.
A. $\frac{2}{3}(2^{3/2} - 1)$ B. $\frac{1}{6}(2^{3/2} - 1)$ C. $\frac{2}{3}(21^{3/2} - 6^{3/2})$ **D.** $\frac{1}{6}(21^{3/2} - 6^{3/2})$

II Advanced (55%)

16. Find the limit $\lim_{x\to 0^+} \sqrt{x}\cos^2\frac{1}{x}$.
A. ∞ B. 1 **C.** 0 D. does not exist

17. Find the limit $\lim_{x\to 0} \frac{\sin(\sin x)}{x}$.
A. ∞ **B.** 1 C. 0 D. does not exist

18. Find the limit $\lim_{x\to \frac{\pi}{4}} \frac{\tan x - 1}{x - \frac{\pi}{4}}$.
A. 2 B. $\sqrt{2}$ C. 1 D. does not exist

19. For what values of a and b will $f(x) = \begin{cases} ax + b, & x \le -1 \\ ax^3 + x + 2b, & x > -1 \end{cases}$ be differentiable for all values of x?
A. $a = -\frac{1}{2}$, $b = 1$ B. $a = \frac{1}{2}$, $b = 1$ C. $a = 1$, $b = -\frac{1}{2}$ D. $a = 1$, $b = \frac{1}{2}$

20. Evaluate the integral $\int_0^{3\pi/2} |\sin x|\, dx$.
A. 1 B. 2 **C.** 3 D. none of the above

21. Find the derivative of $f(x) = \int_2^{1/x} \sin^4 t\, dt$.
A. $\sin^4\left(\frac{1}{x}\right)$ **B.** $-\frac{\sin^4(1/x)}{x^2}$ C. $\cos^4\left(\frac{1}{x}\right)$ D. $-\frac{\cos^4(1/x)}{x^2}$

22. For what values of x is $f(x) = \begin{cases} x^2 + 3, & x \text{ is irrational} \\ 4x, & x \text{ is rational} \end{cases}$ continuous?
A. all rational x B. all real x C. all irrational x **D. none of the above**

23. Consider the function $f(x) = \begin{cases} x\sin\frac{1}{x}, & x \neq 0, \\ 0, & x = 0. \end{cases}$ Which one of the following statements is false?
A. the Intermediate Value Theorem can be applies to f on the interval $[-1, 1]$ **B. the Mean Value Theorem can be applies to f on the interval $[-1, 1]$** C. $-|x| \le f(x) \le |x|$ D. none of the above

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24. Consider the function $f(x) = x^4 - 6x^2 + 1$. Which one of the following statements is false?

A. f is increasing on the interval $[-\sqrt{3}, 0]$ B. $(1, -4)$ is a point of inflection of the graph of f
C. f has no absolute maximum **D. none of the above**

25. Consider the function $f(x) = \dfrac{x^2 - 3}{2x - 4}$. Which one of the following statements is false?

A. $\lim\limits_{x \to \infty} f(x) = \infty$ B. the line $x = 2$ is a vertical asymptote of the graph of f **C. the line $y = \dfrac{1}{2}(x + 1)$ is a slant asymptote of the graph of f** D. none of the above

26. Find $f(4)$ if $\displaystyle\int_0^{x^2} f(t)\, dt = x \cos \pi x$.

A. $\dfrac{1}{4}$ **B.** $-\dfrac{1}{4}$ C. 4 D. -4