

國立高雄大學一〇五學年度第一學期  
理學院微積分基礎能力會考試題 (B)

日期：105 年 12 月 7 日 時間：120 分鐘 學號： 姓名：

Notation:

- $\mathbb{Z}$ : the set of integers (整數集)
- $\mathbb{Q}$ : the set of rational numbers (有理數集)
- $\mathbb{R}$ : the set of real numbers (實數集)
- $\in$ : belong (屬於); for example,  $a \in \mathbb{R}$  means that “ $a$  is a real number.”

I Basic (45%)

1. What is the limit of  $f(x) = 2x + 1$  as  $x$  approaches 0?  
A.  $\lim_{x \rightarrow 0} (2x + 1) = 0$  B.  $\lim_{x \rightarrow 0} (2x + 1) = 1$  C.  $\lim_{x \rightarrow 0} (2x + 1) = 2$  D. none of the above
2. What is the limit of  $f(x) = \frac{x}{|x|}$  as  $x$  approaches 0 from the left?  
A.  $\lim_{x \rightarrow 0^-} \frac{x}{|x|} = -1$  B.  $\lim_{x \rightarrow 0^-} \frac{x}{|x|} = 0$  C.  $\lim_{x \rightarrow 0^-} \frac{x}{|x|} = 1$  D.  $\lim_{x \rightarrow 0^-} \frac{x}{|x|}$  does not exist
3. Which one of the following functions is not continuous on  $\mathbb{R}$ ?  
A.  $f_1(x) = \sin x$  B.  $f_2(x) = \frac{1}{x}$  C.  $f_3(x) = \frac{1}{x^2 + 1}$  D. none of the above
4. Find the horizontal asymptotes (if any) of the function  $f(x) = \sqrt{x^6 + 5x^3} - x^3$ ,  $x \geq 0$ .  
A. no horizontal asymptote B.  $y = -\frac{5}{2}$  C.  $y = \frac{5}{2}$  D.  $y = 0$
5. Find the derivative of  $f(x) = \sqrt{10}$ .  
A.  $\sqrt{10}$  B.  $\frac{1}{\sqrt{10}}$  C.  $\frac{1}{2\sqrt{10}}$  D. none of the above
6. Find an equation for the line that is tangent to the curve  $y = x^3 - x$  at the point  $(-1, 0)$ .  
A.  $y = -\frac{1}{2}(x + 1)$  B.  $y = \frac{1}{2}(x + 1)$  C.  $y = -2(x + 1)$  D.  $y = 2(x + 1)$
7. Find the derivative of  $f(x) = \sin^2(\pi x - 2)$ .  
A.  $2 \sin(\pi x - 2) \cos(\pi x - 2)$  B.  $2\pi \sin(\pi x - 2) \cos(\pi x - 2)$  C.  $2\pi \cos(\pi x - 2)$  D.  $2\pi \sin(\pi x - 2)$
8. Determine the slope of the graph of  $x^2 y^2 - 2x = 4 - 4y$  at the point  $(2, -2)$ .  
A.  $\frac{7}{6}$  B.  $-\frac{7}{6}$  C.  $\frac{6}{7}$  D.  $-\frac{6}{7}$
9. Find all the critical numbers (also called critical points) of  $f(x) = \sin |x|$ .  
A.  $x = \frac{\pi}{2} + n\pi$ ,  $n \in \mathbb{Z}$  B.  $x = 0$ ,  $\frac{\pi}{2} + n\pi$ ,  $n \in \mathbb{Z}$  C.  $x = \pm\left(\frac{\pi}{2} + n\pi\right)$ ,  $n \in \mathbb{N} \cup \{0\}$   
D.  $x = 0$ ,  $\pm\left(\frac{\pi}{2} + n\pi\right)$ ,  $n \in \mathbb{N} \cup \{0\}$
10. Which one of the following functions has no inflection point at  $x = 0$ ?  
A.  $f_1(x) = \sin x$  B.  $f_2(x) = x^{5/3}$  C.  $f_3(x) = x^4$  D. none of the above
11. The graph of the function  $f(x) = \cos x$  on the interval  $(\pi/2, \pi)$  is  
A. increasing, concave upward B. increasing, concave downward C. decreasing, concave upward D. decreasing, concave downward

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12. Let  $y = \sqrt{x}$ . Find the differential  $dy$  of  $y$  at  $x = 4$  with  $dx = 3$ .  
 A.  $\frac{1}{4}$     **B.  $\frac{3}{4}$**     C.  $\sqrt{7} - 2$     D.  $3(\sqrt{7} - 2)$
13. Express the limit  $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{i^4}{n^5}$  as a definite integral.  
**A.  $\int_0^1 x^4 dx$**     B.  $\int_0^1 x^5 dx$     C.  $\int_0^1 \frac{1}{x^4} dx$     D.  $\int_0^1 \frac{1}{x^5} dx$
14. Which one of the following statement is false?  
 A.  $\int_{\pi}^{\pi} \sin x dx = 0$     B.  $\int_0^1 \sqrt{1-x^2} dx = -\int_1^0 \sqrt{1-x^2} dx$     **C.  $\int_{-1}^1 \frac{1}{x^2} dx = \left[-\frac{1}{x}\right]_{-1}^1 = -1 - 1 = -2$**     D. none of the above
15. Evaluate the definite integral  $\int_1^2 x^3 \sqrt{x^4 + 5} dx$ .  
 A.  $\frac{2}{3}(2^{3/2} - 1)$     B.  $\frac{1}{6}(2^{3/2} - 1)$     C.  $\frac{2}{3}(21^{3/2} - 6^{3/2})$     **D.  $\frac{1}{6}(21^{3/2} - 6^{3/2})$**

## II Advanced (55%)

16. Find the limit  $\lim_{x \rightarrow 0^+} \sqrt{x} \cos^2 \frac{1}{x}$ .  
 A.  $\infty$     B. 1    **C. 0**    D. does not exist
17. Find the limit  $\lim_{x \rightarrow 0} \frac{\sin(\sin x)}{x}$ .  
 A.  $\infty$     **B. 1**    C. 0    D. does not exist
18. Find the limit  $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan x - 1}{x - \frac{\pi}{4}}$ .  
**A. 2**    B.  $\sqrt{2}$     C. 1    D. does not exist
19. For what values of  $a$  and  $b$  will  $f(x) = \begin{cases} ax + b, & x \leq -1 \\ ax^3 + x + 2b, & x > -1 \end{cases}$  be differentiable for all values of  $x$ ?  
**A.  $a = -\frac{1}{2}, b = 1$**     B.  $a = \frac{1}{2}, b = 1$     C.  $a = 1, b = -\frac{1}{2}$     D.  $a = 1, b = \frac{1}{2}$
20. Evaluate the integral  $\int_0^{3\pi/2} |\sin x| dx$ .  
 A. 1    B. 2    **C. 3**    D. none of the above
21. Find the derivative of  $f(x) = \int_2^{1/x} \sin^4 t dt$ .  
 A.  $\sin^4\left(\frac{1}{x}\right)$     **B.  $-\frac{\sin^4(1/x)}{x^2}$**     C.  $\cos^4\left(\frac{1}{x}\right)$     D.  $-\frac{\cos^4(1/x)}{x^2}$
22. For what values of  $x$  is  $f(x) = \begin{cases} \frac{x^2 - x - 6}{x - 3}, & x \neq 3 \\ 5, & x = 3 \end{cases}$  continuous?  
 A. all real  $x$  except 2    B. all real  $x$  except 3    C. all real  $x$  except 5    **D. none of the above**
23. Find  $\frac{d^{2016}}{dx^{2016}}(\cos x)$ .  
 A.  $\sin x$     **B.  $\cos x$**     C.  $-\sin x$     D.  $-\cos x$

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24. Consider the function  $f(x) = x^4 - 6x^2 + 1$ . Which one of the following statements is false?  
A.  $f$  is increasing on the interval  $[-\sqrt{3}, 0]$     B.  $f$  is decreasing on the interval  $[0, \sqrt{3}]$     C.  $f$  is increasing on the interval  $[\sqrt{3}, \infty)$     **D. none of the above**
25. Consider the function  $f(x) = \frac{x^2 - 3}{2x - 4}$ . Which one of the following statements is false?  
A.  $\lim_{x \rightarrow \infty} f(x) = \infty$     B.  $\lim_{x \rightarrow 2^+} f(x) = \infty$     **C.  $\lim_{x \rightarrow 2^-} f(x) = \infty$**     D. none of the above
26. Suppose that  $f$  has a positive derivative for all values of  $x$  and that  $f(1) = 0$ . Let  $g(x) = \int_0^x f(t) dt$ . Which one of the following statements is false?  
**A.  $g$  has a relative maximum at  $x = 1$**     B.  $g$  is a differentiable function of  $x$     C. the graph of  $g$  has a horizontal tangent at  $x = 1$     D. none of the above