

國立高雄大學一〇五學年度第一學期
理學院微積分基礎能力會考試題 (C)

日期：105 年 12 月 7 日 時間：120 分鐘 學號： 姓名：

Notation:

- \mathbb{Z} : the set of integers (整數集)
- \mathbb{Q} : the set of rational numbers (有理數集)
- \mathbb{R} : the set of real numbers (實數集)
- \in : belong (屬於); for example, $a \in \mathbb{R}$ means that “ a is a real number.”

I Basic (45%)

1. What is the limit of $f(x) = 2x + 1$ as x approaches 0?
A. $\lim_{x \rightarrow 0} (2x + 1) = 0$ B. $\lim_{x \rightarrow 0} (2x + 1) = 1$ C. $\lim_{x \rightarrow 0} (2x + 1) = 2$ D. none of the above
2. What is the limit of $f(x) = \frac{x}{|x|}$ as x approaches 0 from the left?
A. $\lim_{x \rightarrow 0^-} \frac{x}{|x|} = -1$ B. $\lim_{x \rightarrow 0^-} \frac{x}{|x|} = 0$ C. $\lim_{x \rightarrow 0^-} \frac{x}{|x|} = 1$ D. $\lim_{x \rightarrow 0^-} \frac{x}{|x|}$ does not exist
3. Which one of the following functions is not continuous on \mathbb{R} ?
A. $f_1(x) = x^2 + x + 1$ B. $f_2(x) = \frac{1}{x}$ C. $f_3(x) = \frac{1}{x^2 + 1}$ D. none of the above
4. Find the horizontal asymptotes (if any) of the function $f(x) = \sqrt{x^6 + 5x^3} - x^3$, $x \geq 0$.
A. no horizontal asymptote B. $y = -\frac{5}{2}$ C. $y = \frac{5}{2}$ D. $y = 0$
5. Find the derivative of $f(x) = \sqrt{10}$.
A. $\sqrt{10}$ B. $\frac{1}{\sqrt{10}}$ C. $\frac{1}{2\sqrt{10}}$ D. none of the above
6. Find an equation for the line that is tangent to the curve $y = x^3 - x$ at the point $(-1, 0)$.
A. $y = -\frac{1}{2}(x + 1)$ B. $y = \frac{1}{2}(x + 1)$ C. $y = -2(x + 1)$ D. $y = 2(x + 1)$
7. Find the derivative of $f(x) = \sqrt{\frac{x-1}{x+1}}$.
A. $\frac{1}{2(x-1)^{1/2}(x+1)^{3/2}}$ B. $\frac{1}{(x-1)^{1/2}(x+1)^{3/2}}$ C. $\frac{1}{2}\sqrt{\frac{x+1}{x-1}}$ D. $\sqrt{\frac{x+1}{x-1}}$
8. Determine the slope of the graph of $x^2y^2 - 2x = 4 - 4y$ at the point $(2, -2)$.
A. $\frac{7}{6}$ B. $-\frac{7}{6}$ C. $\frac{6}{7}$ D. $-\frac{6}{7}$
9. Find all the critical numbers (also called critical points) of $f(x) = x^{1/3}(x - 4)$.
A. $x = 0$ B. $x = 0, 4$ C. $x = 0, 1, 4$ D. $x = 0, 1$
10. Which one of the following functions has no inflection point at $x = 0$?
A. $f_1(x) = x^{1/3}$ B. $f_2(x) = x^{5/3}$ C. $f_3(x) = x^4$ D. none of the above
11. The graph of the function $f(x) = x^4 - 4x^3 + 10$ on the interval $(2, 3)$ is
A. increasing, concave upward B. increasing, concave downward C. decreasing, concave upward D. decreasing, concave downward
12. Let $y = \sqrt{x}$. Find the differential dy of y at $x = 4$ with $dx = 3$.
A. $\frac{1}{4}$ B. $\frac{3}{4}$ C. $\sqrt{7} - 2$ D. $3(\sqrt{7} - 2)$

國立高雄大學一〇五學年度第一學期
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13. Consider the function $f(x) = \begin{cases} \frac{x^2 - x}{x^2 - 1}, & x < 1, \\ \frac{1}{2}, & x = 1, \\ \frac{x^3 - 1}{x^2 - 1}, & x > 1. \end{cases}$ Which one of the following statement is true?
A. f is continuous from the left at $x = 1$ B. f is continuous from the right at $x = 1$
 C. f is continuous at $x = 1$ D. none of the above
14. Consider the function $f(x) = x^3 - 3x^2 + 1, -\frac{1}{2} \leq x < 4$. Which one of the following statement is false?
 A. f has two critical points B. f has an absolute minimum **C. f has an absolute maximum** D. none of the above
15. Consider the function $f(x) = -\frac{8}{x^2 - 4}$. Which one of the following statement is false?
 A. $\lim_{x \rightarrow -2^-} f(x) = -\infty$ B. $\lim_{x \rightarrow 2^-} f(x) = \infty$ C. f has two vertical asymptotes **D. none of the above**

II Advanced (55%)

16. Find the limit $\lim_{x \rightarrow 2^-} \frac{x^2 + x - 6}{|x - 2|}$.
 A. ∞ B. 5 **C. -5** D. does not exist
17. Find the limit $\lim_{x \rightarrow 2} \frac{\sqrt{x^2 + 12} - 4}{x - 2}$.
 A. 1 **B. $\frac{1}{2}$** C. 0 D. does not exist
18. Find the limit $\lim_{x \rightarrow 1} \left(\frac{1}{x - 1} + \frac{1}{x^2 - 3x + 2} \right)$.
A. -1 B. 0 C. 1 D. does not exist
19. For what values of a and b will $f(x) = \begin{cases} ax + b, & x \leq -1 \\ ax^3 + x + 2b, & x > -1 \end{cases}$ be differentiable for all values of x ?
A. $a = -\frac{1}{2}, b = 1$ B. $a = \frac{1}{2}, b = 1$ C. $a = 1, b = -\frac{1}{2}$ D. $a = 1, b = \frac{1}{2}$
20. If $f(x) = \sqrt{x}g(x)$, where $g(4) = 8$ and $g'(4) = 7$, find $f'(4)$.
 A. 14 B. 15 **C. 16** D. none of the above
21. Which one of the following statements is false?
 A. If $f'(x)$ exists and is nonzero for all x , then $f(1) \neq f(0)$ **B. If f is even, then f' is even** C. If f is increasing and $f(x) > 0$ on an interval I , then $g(x) = 1/f(x)$ is decreasing on I D. none of the above
22. For what values of x is $f(x) = \begin{cases} \frac{x^2 - x - 6}{x - 3}, & x \neq 3 \\ 5, & x = 3 \end{cases}$ continuous?
 A. all real x except 2 B. all real x except 3 C. all real x except 5 **D. none of the above**
23. Find the limit $\lim_{x \rightarrow 1} \frac{x^{2016} - 1}{x - 1}$.
 A. 2015 **B. 2016** C. 2017 D. does not exist

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24. Consider the function $f(x) = x^4 - 6x^2 + 1$. Which one of the following statements is false?
A. f is increasing on the interval $[-\sqrt{3}, 0]$ B. f is decreasing on the interval $[0, \sqrt{3}]$ C. f is increasing on the interval $[\sqrt{3}, \infty)$ **D. none of the above**
25. Consider the function $f(x) = \frac{x^2 - 3}{2x - 4}$. Which one of the following statements is false?
A. $\lim_{x \rightarrow \infty} f(x) = \infty$ B. $\lim_{x \rightarrow 2^+} f(x) = \infty$ **C. $\lim_{x \rightarrow 2^-} f(x) = \infty$** D. none of the above
26. Find $f'(x)$ if it is known that $\frac{d}{dx}[f(2x)] = x^2$.
A. $\frac{x^2}{8}$ B. $\frac{x^2}{4}$ C. $\frac{x^2}{2}$ D. none of the above