

國立高雄大學一〇五學年度第二學期  
理學院微積分基礎能力會考試題 (A)

系級：

學號：

姓名：

### I Basic (60%)

1. Which one of the following statements is false?

A.  $\int_{-\pi}^{\pi} \sin x \sin 2x \, dx = 0$     B.  $\int_{-\pi}^{\pi} \sin x \cos 2x \, dx = 0$     C.  $\int_{-\pi}^{\pi} \cos^2 2x \, dx = \pi$   
**D.  $\int_{-\pi}^{\pi} \sin 2x \cos 2x \, dx = \pi$**

2. Find  $\int \sin^4 x \cos^2 x \, dx$ .

A.  $-\frac{x}{16} - \frac{1}{48} \sin 4x - \frac{1}{64} \cos^3 2x + C$     B.  $\frac{x}{16} + \frac{1}{64} \sin 4x - \frac{1}{48} \cos^3 2x + C$     C.  $-\frac{x}{16} + \frac{1}{48} \sin 4x - \frac{1}{64} \sin^3 2x + C$   
**D.  $\frac{x}{16} - \frac{1}{64} \sin 4x - \frac{1}{48} \sin^3 2x + C$**

3. Consider the following series. Let  $R$  denote the radius of convergence of the corresponding power series. Which one of the following statements is false?

A.  $\sum_{n=0}^{\infty} (-1)^n \frac{x^n}{n+1}, R=1$     **B.  $\sum_{n=0}^{\infty} \frac{(2n)!x^{2n}}{n!}, R=\frac{1}{2}$**     C.  $\sum_{n=0}^{\infty} \frac{x^{5n}}{n!}, R=\infty$     D.  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-4)^n}{n9^n}, R=9$

4. Which improper integral is convergent?

A.  $\int_0^{\pi} \tan x \, dx$     B.  $\int_{-\infty}^{\infty} \arctan x \, dx$     **C.  $\int_0^1 \frac{1}{\sqrt{1-x^2}} \, dx$**     D.  $\int_2^{\infty} \frac{1}{x \ln x} \, dx$

5. Find  $\int x e^{3x} \, dx$ .

**A.  $\frac{x}{3} e^{3x} - \frac{1}{9} e^{3x} + C$**     B.  $\frac{x}{9} e^{3x} - \frac{1}{3} e^{3x} + C$     C.  $\frac{x}{3} e^{3x} + \frac{1}{9} e^{3x} + C$     D.  $\frac{x}{9} e^{3x} + \frac{1}{3} e^{3x} + C$

6. Evaluate the integral  $\int_1^{\sqrt{2}} \frac{dx}{x^2 \sqrt{4-x^2}}$ .

A.  $\frac{\sqrt{3}+1}{4}$     B.  $\frac{\sqrt{3}+1}{2}$     **C.  $\frac{\sqrt{3}-1}{4}$**     D.  $\frac{\sqrt{3}-1}{2}$

7. Evaluate the limit  $\lim_{x \rightarrow 1^+} \left( \frac{1}{\ln x} - \frac{1}{x-1} \right)$ .

A.  $\infty$     **B.  $\frac{1}{2}$**     C. 1    D. 2

8. Evaluate the integral  $\int_1^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}} \, dx$ .

A. divergent    B. 0    C.  $2e$     **D.  $2e^{-1}$**

9. Which one of the following series is convergent?

A.  $\sum_{k=1}^{\infty} \frac{\arctan k}{k}$     **B.  $\sum_{k=2}^{\infty} \frac{k!}{k^k}$**     C.  $\sum_{k=5}^{\infty} \left( \frac{\pi}{e} \right)^{k-1}$     D.  $\sum_{k=1}^{\infty} \ln \left( \frac{k}{k+1} \right)$

10. Evaluate the integral  $\int_0^{2/\sqrt{3}} \frac{1}{4+9x^2} \, dx$ .

**A.  $\frac{\pi}{18}$**     B.  $\frac{\pi}{36}$     C.  $18\pi$     D.  $36\pi$

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11. Evaluate the integral  $\int_0^{\pi/4} \sec^4 \theta \tan^4 \theta d\theta$ .  
 A.  $\frac{1}{7}$    B.  $\frac{1}{5}$    **C.  $\frac{12}{35}$**    D.  $\frac{2}{35}$
12. Evaluate the integral  $\int_1^2 x^3 \ln x dx$ .  
 A.  $8 \ln 2 - \frac{15}{16}$    **B.  $4 \ln 2 - \frac{15}{16}$**    C.  $8 \ln 2 + \frac{15}{16}$    D.  $4 \ln 2 + \frac{15}{16}$
13. Which one of the following series is divergent?  
 A.  $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$    B.  $\sum_{n=3}^{\infty} \frac{n^2}{e^n}$    **C.  $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$**    D.  $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$
14. Find  $\int \frac{x^2 + x + 3}{x^4 + 6x^2 + 9} dx$ .  
 A.  $\frac{1}{3} \arctan \frac{x}{\sqrt{3}} - \frac{1}{x^2 + 3} + C$    B.  $\frac{1}{\sqrt{3}} \arctan \frac{x}{3} - \frac{1}{2(x^2 + 3)} + C$    **C.  $\frac{1}{\sqrt{3}} \arctan \frac{x}{\sqrt{3}} - \frac{1}{2(x^2 + 3)} + C$**   
 D.  $\frac{1}{3} \arctan \frac{x}{3} - \frac{1}{x^2 + 3} + C$
15. Evaluate  $\sum_{k=1}^{\infty} \frac{5}{2^k}$ .  
**A. 5**   B.  $\frac{5}{2}$    C. 1   D.  $\infty$

## II Advanced (40%)

16. Let  $a_1 = 2$ ,  $a_2 = 2 + \frac{1}{2}$ ,  $a_3 = 2 + \frac{1}{2 + \frac{1}{2}}$ ,  $a_4 = 2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}}$ ,  $\dots$ . Find the limit  $\lim_{n \rightarrow \infty} a_n$ .  
 A.  $\sqrt{2} - 1$    B.  $\sqrt{2}$    **C.  $\sqrt{2} + 1$**    D.  $\sqrt{2} + 2$
17. Evaluate the integral  $\int_e^{e^2} (\ln x)^{\ln x} \left( \frac{1}{x} + \frac{\ln(\ln x)}{x} \right) dx$ .  
**A. 3**   B. 2   C. 1   D. 0
18. Find the sum of the series  $\sum_{k=2}^{\infty} \frac{\sqrt{k+1} - \sqrt{k}}{\sqrt{k^2 + k}}$ .  
 A. 1   B.  $2\sqrt{2}$    C.  $\sqrt{2}$    **D.  $\frac{\sqrt{2}}{2}$**
19. Find the limit  $\lim_{n \rightarrow \infty} \sqrt[n]{n}$ .  
**A. 1**   B. 0   C.  $e$    D.  $\infty$
20. Which one of the following series is divergent?  
 A.  $\sum_{n=0}^{\infty} \frac{\sin(n + \frac{1}{2})\pi}{1 + \sqrt{n}}$    **B.  $\sum_{n=1}^{\infty} (-1)^{n-1} e^{2/n}$**    C.  $\sum_{n=1}^{\infty} (-1)^n (\sqrt{n+1} - \sqrt{n})$    D.  $\sum_{n=1}^{\infty} (-1)^n \sin\left(\frac{\pi}{n}\right)$
21. If  $f(x) = \cos \sqrt{x}$ , then  $f^{(4)}(0) =$ .  
 A.  $-\frac{4!}{8!}$    **B.  $\frac{4!}{8!}$**    C.  $-\frac{1}{8!}$    D.  $\frac{1}{8!}$

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22. Let  $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-2)^n}{n}$ . Find the interval of convergence for  $\int f(x) dx$ .

- A. (1, 3)   B. [1, 3)   C. (1, 3]   **D. [1, 3]**

23. Which one of the following series is convergent?

- A.  $\sum_{n=1}^{\infty} \frac{n!}{10^n}$    **B.  $\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^{n^2}$**    C.  $\sum_{n=1}^{\infty} \left(\ln\left(e^2 + \frac{1}{n}\right)\right)^{n+1}$    D.  $\sum_{n=1}^{\infty} \frac{3^n}{n^3 2^n}$

24. Which one of the following series is conditional convergent?

- A.  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^3 + 1}$    B.  $\sum_{n=1}^{\infty} (-1)^n \frac{\arctan n}{n^2 + 1}$    C.  $\sum_{n=1}^{\infty} (-1)^n \frac{(n!)^2 3^n}{(2n+1)!}$    **D.  $\sum_{n=1}^{\infty} (-1)^n \ln\left(1 + \frac{1}{n}\right)$**

25. Evaluate the integral  $\int_{-\pi/4}^{\pi/4} \frac{x^2 \tan x}{1 + \cos^4 x} dx$ .

- A. 1   B.  $-\pi$    **C. 0**   D.  $\pi$