國立高雄大學一〇五學年度第二學期
理學院微積分基礎能力會考試題 (B)

系級： 學號： 姓名：

I Basic (68%)

1. Evaluate $\arccos\left(-\frac{1}{2}\right)$.

   A. π B. $\frac{\pi}{2}$ C. $\frac{3\pi}{4}$ **D.** $\frac{2\pi}{3}$

2. The domain of $\arctan x$ is

   A. $[-1,1]$ B. $(-\infty,-1]\cup[1,\infty)$ C. $(0,\pi)$ **D.** $(-\infty,\infty)$

3. Find the derivative of the function $f(x)=\arctan(\ln x)$.

   A. $\frac{1}{1+(\ln x)^2}$ **B.** $\frac{1}{x[1+(\ln x)^2]}$ C. $\frac{1}{(1+x^2)\arctan\frac{1}{x}}$ D. $\frac{1}{[1+(\ln x)^2]\arctan\frac{1}{x}}$

4. Evaluate the integral $\int_0^{\pi/2}\frac{\sin x}{1+\cos^2 x}\,dx$.

   A. $\frac{\pi}{2}$ B. $-\frac{\pi}{2}$ **C.** $\frac{\pi}{4}$ D. $-\frac{\pi}{4}$

5. Which one of the following formulas is false?

   A. $\sinh^2 x+\cosh^2 x=1$ B. $\sinh(-x)=-\sinh x$ C. $\sinh(x+y)=\sinh x\cosh y+\cosh x\sinh y$ D. $\sinh 2x=2\sinh x\cosh x$

6. Evaluate the integral $\int_0^1\frac{dt}{\sqrt{16t^2+1}}\,dt$.

   A. $\frac{1}{4}\ln(1+\sqrt{2})$ B. $\ln(1+\sqrt{2})$ **C.** $\frac{1}{4}\ln(4+\sqrt{17})$ D. $\ln(4+\sqrt{17})$

7. Solve the differential equation $(1+x^2)y'-2xy=0$.

   A. $y=Cx$ **B.** $y=C(1+x^2)$ C. $y=Ce^{1+x^2}$ D. $y=e^{C(1+x^2)}$

8. Given the initial condition $y(1)=2$, find the particular solution of the equation $2xy'-\ln x^2=0$.

   A. $y=\ln x+2$ B. $y=(\ln x)^2+2$ C. $y=\frac{1}{2}\ln x+1$ **D.** $y=\frac{1}{2}(\ln x)^2+2$

9. Solve the differential equation $y'-y=e^{2x}$.

   A. $y=C(e^{2x}+e^x)$ **B.** $y=e^{2x}+Ce^x$ C. $y=Ce^{2x}+e^x$ D. $y=Ce^{2x}-e^x$

10. Find the area bounded by the graphs of $x=y^2$ and $x=2-y^2$.

    A. $\frac{8}{3}$ B. $\frac{4}{3}$ C. $\frac{10}{3}$ D. 3

11. Find the volume of the solid generated by revolving the region bounded by the graphs of $x=\sqrt{2\sin 2y}$, $x=0$, $y=0$, and $y=\frac{\pi}{2}$ about the y-axis.

    A. 4π B. $\sqrt{2}\pi$ **C.** 2π D. π

12. Find the arc length of the graph of $y^2=x^3$ on the interval $[1,4]$.

    A. $\frac{1}{27}(40\sqrt{10}-13\sqrt{13})$ **B.** $\frac{1}{27}(80\sqrt{10}-13\sqrt{13})$ C. $\frac{1}{9}(80\sqrt{5}+13\sqrt{13})$ D. $\frac{1}{9}(40\sqrt{5}+13\sqrt{13})$

13. Find the area of the surface formed by revolving the graph of $y = \frac{x^3}{6} + \frac{1}{2x}$ on the interval $[1, 2]$ about the x-axis.

A. $\frac{49\pi}{16}$ B. 3π **C.** $\frac{47\pi}{16}$ D. $\frac{25\pi}{8}$

14. The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the x-axis to generate a solid. Find the volume of the solid.

A. $\frac{119\pi}{5}$ B. $\frac{118\pi}{5}$ **C.** $\frac{117\pi}{5}$ D. $\frac{116\pi}{5}$

15. Find the volume of a right pyramid whose altitude is 5 and whose base is a square with sides of length 3.

A. 15 B. 25 C. 45 D. 9

16. The general solution of the logistic differential equation $\frac{dy}{dt} = ay\left(1 - \frac{y}{b}\right)$ is

A. $y = \frac{a}{1 + e^{Cbt}}$ B. $y = \frac{Cb}{1 + e^{-at}}$ **C.** $y = \frac{b}{1 + Ce^{-at}}$ D. $y = \frac{a}{C + e^{-bt}}$

17. Which one of the following equations is not a first-order linear differential equation?

A. $y' + y^2 = x$ B. $y' + xy = e^x y$ C. $(2 + x^2)y' - xy = e^{x^2}$ D. $y' + (\sin x)y = x^x$

II Advanced (32%)

18. Find $\int x \coth^{-1} x \, dx$.

A. $-\frac{x^2 - 1}{2} \coth^{-1} x + x + C$ B. $-\frac{x^2 - 1}{4} \coth^{-1} x - \frac{x}{2} + C$ C. $(x^2 - 1) \coth^{-1} x - \frac{x}{4} + C$
D. $\frac{x^2 - 1}{2} \coth^{-1} x + \frac{x}{2} + C$

19. Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$.

A. $\frac{10}{3}$ B. $\frac{16}{3}$ C. $\frac{4\sqrt{2}}{3}$ D. 2

20. Find the volume of the solid generated by revolving the region in the first quadrant bounded above by the curve $y = x^2$, below by the x-axis, and on the right by the line $x = 1$, about the line $x = -1$.

A. $\frac{4\pi}{3}$ **B.** $\frac{7\pi}{6}$ C. π D. $\frac{5\pi}{6}$

21. Find the volume of the solid generated by revolving the region bounded by the graphs of $x = 1 + (y - 2)^2$ and $x = 2$ about the x-axis.

A. $\frac{17\pi}{3}$ **B.** $\frac{16\pi}{3}$ C. 5π D. $\frac{14\pi}{3}$

22. If the curve $y = f(x)$, $a \le x \le b$, is rotated about the horizontal line $y = c$, where $f(x) \le c$, then the formula for the area of the resulting surface is

A. $2\pi \int_a^b x\sqrt{1 + [f'(x)]^2} \, dx$ B. $2\pi \int_a^b (c - x)\sqrt{1 + [f'(x)]^2} \, dx$ C. $2\pi \int_a^b f(x)\sqrt{1 + [f'(x)]^2} \, dx$
D. $2\pi \int_a^b [c - f(x)]\sqrt{1 + [f'(x)]^2} \, dx$

23. Find $\int (\arcsin x)^2 \, dx$.

A. $x(\arcsin x)^2 + 3x + 2\sqrt{1 - x^2} \arcsin x + C$ **B.** $x(\arcsin x)^2 - 2x + 2\sqrt{1 - x^2} \arcsin x + C$
C. $2x(\arcsin x)^2 - 2x + 4\sqrt{1 - x^2} \arcsin x + C$ D. $3x(\arcsin x)^2 + x + \sqrt{1 - x^2} \arcsin x + C$

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24. The Bernoulli equation is an equation of the form $y' + P(x)y = Q(x)y^n$. The general solution of a Bernoulli equation is

$$y^{1-n}e^{\int(1-n)P(x)dx} = \int (1-n)Q(x)e^{\int(1-n)P(x)dx}\,dx + C.$$

Use the formula to solve the equation $y' + xy = xy^{-1}$.

A. $y^2 = C(1 + e^{-x^2})$ B. $y^2 = 1 + Cxe^{-x^2}$ C. $y^2 = e^{x^2} + C$ **D.** $y^2 = 1 + Ce^{-x^2}$

25. Which one of the following equalities is false?

A. $\tan\left(\operatorname{arcsec}\frac{x}{3}\right) = \frac{\sqrt{x^2-9}}{3}$ B. $\operatorname{arccsc} x = \arcsin\frac{1}{x}$ for $x \geq 1$ **C.** $\arccos(-x) = \arccos x$ **for** $|x| \leq 1$ D. $\arcsin(-x) = -\arcsin x$ for $|x| \leq 1$