

國立高雄大學一〇五學年度第二學期
理學院微積分基礎能力會考試題 (C)

系級：

學號：

姓名：

I Basic (60%)

1. Find $\int \left(3\sqrt{x} - \frac{1}{x^4}\right) dx$.
 A. $3x^{3/2} + \frac{1}{4x^3} + C$ B. $2x^{3/2} - \frac{1}{3x^3} + C$ C. $\frac{3}{2}x^{3/2} - \frac{1}{4x^3} + C$ **D. $2x^{3/2} + \frac{1}{3x^3} + C$**
2. Evaluate the integral $\int_0^2 x\sqrt{x^2+1} dx$.
 A. $\frac{5}{2}\sqrt{5} + \frac{5}{2}$ B. $\frac{5}{2}\sqrt{5} - \frac{5}{2}$ C. $\frac{5}{3}\sqrt{5} + \frac{1}{3}$ **D. $\frac{5}{3}\sqrt{5} - \frac{1}{3}$**
3. Find $\int \frac{1}{1+e^{-x}} dx$.
 A. $\frac{1}{1+e^{-x}} + C$ **B. $\ln(e^x + 1) + C$** C. $\ln(e^{-x} + 1) + C$ D. $-\ln(e^{-x} + 1) + C$
4. Which improper integral is convergent?
 A. $\int_0^2 \frac{x}{x^2-1} dx$ B. $\int_1^\infty \ln x dx$ **C. $\int_{-\infty}^0 \frac{x}{(1+x^2)^2} dx$** D. $\int_2^\infty \frac{1}{x \ln x} dx$
5. Find $\int xe^{3x} dx$.
A. $\frac{x}{3}e^{3x} - \frac{1}{9}e^{3x} + C$ B. $\frac{x}{9}e^{3x} - \frac{1}{3}e^{3x} + C$ C. $\frac{x}{3}e^{3x} + \frac{1}{9}e^{3x} + C$ D. $\frac{x}{9}e^{3x} + \frac{1}{3}e^{3x} + C$
6. Evaluate the trigonometric function $\cos \frac{5\pi}{4}$.
 A. $-\frac{\sqrt{3}}{2}$ B. $\frac{\sqrt{3}}{2}$ **C. $-\frac{\sqrt{2}}{2}$** D. $\frac{\sqrt{2}}{2}$
7. Which one of the following statements is false?
 A. The domain of $\csc x$ is all $x \neq n\pi$ **B. The period of $\tan x$ is 2π** C. The range of $\sec x$ is $(-\infty, -1] \cup [1, \infty)$ D. $\lim_{x \rightarrow 0^+} \cot x = \infty$
8. Evaluate the integral $\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx$.
 A. divergent B. 0 C. $2e$ **D. $2e^{-1}$**
9. Assume $0 \leq x < 2\pi$. Solve all solutions of the equation $\sin^2 x + \cos x - 1 = 0$.
 A. $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ **B. $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$** C. $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ D. $x = 0, \frac{\pi}{3}, \pi, \frac{2\pi}{3}$
10. Find the derivative of $f(x) = \tan \sqrt{x^2+1}$.
A. $\frac{x}{\sqrt{x^2+1}} \sec^2 \sqrt{x^2+1}$ B. $\frac{1}{2\sqrt{x^2+1}} \sec^2 \sqrt{x^2+1}$ C. $\sec^2 x$ D. $\sec^2 \sqrt{x^2+1}$
11. Evaluate the integral $\int_0^{\pi^2} \frac{\cos \sqrt{t}}{\sqrt{t}} dt$.
A. 0 B. π C. $\sin(-1) - \sin 1$ D. $\sin 1 - \sin(-1)$

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12. Evaluate the integral $\int_1^2 x^3 \ln x \, dx$.
 A. $8 \ln 2 - \frac{15}{16}$ **B. $4 \ln 2 - \frac{15}{16}$** C. $8 \ln 2 + \frac{15}{16}$ D. $4 \ln 2 + \frac{15}{16}$
13. Find the slope of the tangent line to the curve $\tan(x + y) = x$ at the point $(0, 0)$.
 A. -1 B. 1 **C. 0** D. ∞
14. Evaluate the limit $\lim_{x \rightarrow 0^-} \frac{\tan x}{x^2}$.
 A. 0 B. 1 **C. $-\infty$** D. ∞
15. Assume that $\int_2^6 f(x) \, dx = 5$ and $\int_2^6 g(x) \, dx = 3$. Which one of the following statements is false?
A. $\int_2^6 f(x)g(x) \, dx = 15$ B. $\int_6^6 [2f(x) + g(x)] \, dx = 0$ C. $\int_2^6 [2f(x) - 3g(x)] \, dx = 1$
 D. $\int_6^2 5f(x) \, dx = -25$

II Advanced (40%)

16. Let $f(x) = \sin x$. Find $f^{(2017)}(x)$.
 A. $\sin x$ B. $-\sin x$ **C. $\cos x$** D. $-\cos x$
17. Evaluate the integral $\int_e^{e^2} (\ln x)^{\ln x} \left(\frac{1}{x} + \frac{\ln(\ln x)}{x} \right) dx$.
A. 3 B. 2 C. 1 D. 0
18. Use the identity $\sin \alpha \cos \beta = \frac{1}{2}[\sin(\alpha - \beta) + \sin(\alpha + \beta)]$ to find $\int \sin 3x \cos 5x \, dx$.
 A. $\frac{\sin 8x}{16} + \frac{\sin 2x}{4} + C$ B. $-\frac{\sin 8x}{16} + \frac{\sin 2x}{4} + C$ C. $-\frac{\cos 8x}{16} - \frac{\cos 2x}{4} + C$ **D. $-\frac{\cos 8x}{16} + \frac{\cos 2x}{4} + C$**
19. Use the identity $\sin^2 x + \cos^2 x = 1$ to evaluate the integral $\int_0^{\pi/2} \cos^5 x \, dx$.
A. $\frac{8}{15}$ B. $\frac{2}{15}$ C. $\frac{28}{15}$ D. $\frac{22}{15}$
20. Evaluate the limit $\lim_{x \rightarrow 1^+} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right)$.
 A. ∞ **B. $\frac{1}{2}$** C. 1 D. 2
21. Evaluate the integral $\int_0^{\pi/2} e^x \cos x \, dx$.
 A. $-\frac{1}{2}(e^{\pi/2} - 1)$ **B. $\frac{1}{2}(e^{\pi/2} - 1)$** C. $-(e^{\pi/2} - 1)$ D. $e^{\pi/2} - 1$
22. Evaluate the limit $\lim_{x \rightarrow 0^+} \left(1 + \frac{3}{x} \right)^x$.
 A. e B. e^3 C. 0 **D. 1**
23. Evaluate the integral $\int_1^e (\ln x)^2 \, dx$.
 A. $e - 1$ **B. $e - 2$** C. $2e - 1$ D. $2e - 2$

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24. Consider the function $f(x) = \sin x + \tan x$. Which one of the following statements is false?
A. The domain of f is all $x \neq \frac{\pi}{2} + n\pi$ B. The range of f is $(-\infty, \infty)$ C. $f'(x) > 0$ if $\pi < x < \frac{3\pi}{2}$ **D. The period of f is π**
25. Consider the function $f(x) = \sin x + \cos x$. Which one of the following statements is false?
(use the identity $\sin \alpha \cos \beta = \frac{1}{2}[\sin(\alpha - \beta) + \sin(\alpha + \beta)]$)
A. f is a periodic function with period 2π B. $f'(x) < 0$ if $\frac{\pi}{4} < x < \frac{5\pi}{4}$ **C. The amplitude of f is $\sqrt{3}$** D. The domain of f is $(-\infty, \infty)$