國立高雄大學理學院 108 學年度第 2 學期
微積分基礎能力會考試題

◎ 單選擇題 (單選十題，每題四分，共四十分，答錯不倒扣)

(1) Evaluate the integral $\int_0^t e^s \sin(t-s)\,ds$.

(A) $\frac{1}{2}(e^t \sin t + e^t \cos t - 1)$ (B) $\frac{1}{4}(e^t \sin t + e^t \cos t - 1)$

(C) $\frac{1}{2}(e^t - \cos t - \sin t)$ (D) $\frac{1}{4}(e^t - \cos t - \sin t)$

(2) Evaluate the integral $\int_0^{\pi} \sqrt{1+\cos 2x}\,dx$.

(A) 0 (B) 1 (C) $\sqrt{2}$ (D) $2\sqrt{2}$

(3) Evaluate the integral $\int_0^{3\sqrt{3}/2} \frac{x^3}{(4x^2+9)^{3/2}}\,dx$.

(A) $\frac{3}{32}$ (B) $-\frac{3}{32}$ (C) $\frac{1}{16}$ (D) $-\frac{1}{16}$

(4) Evaluate the integral $\int_0^1 \frac{x-4}{x^2-5x+6}\,dx$.

(A) $3\ln 2 - \ln 3$ (B) $-3\ln 2 + \ln 3$ (C) $3\ln 3 - \ln 2$ (D) $-3\ln 3 + \ln 2$

(5) Find the length of the arc of the curve $y = \arcsin x + \sqrt{1-x^2}$ from $(0,1)$ to $\left(\frac{\sqrt{3}}{2}, \frac{\pi}{3} + \frac{1}{2}\right)$.

(A) $2\left(\sqrt{2-\sqrt{3}} + \sqrt{2}\right)$ (B) $2\left(\sqrt{2-\sqrt{3}} - \sqrt{2}\right)$

(C) $2\left(\sqrt{2+\sqrt{3}} + \sqrt{2}\right)$ (D) $2\left(\sqrt{2+\sqrt{3}} - \sqrt{2}\right)$

(6) Which one of the following statements is false?

(A) The sequence $\left\{\frac{\ln n}{\ln 2n}\right\}$ converges to 1.

(B) The sequence $\left\{\frac{n!}{2^n}\right\}$ is bounded.

(C) The series $\sum_{n=1}^{\infty} \ln\left(\frac{n^2+1}{2n^2+1}\right)$ is divergent.

(D) The series $\sum_{n=1}^{\infty} \left(\cos\frac{1}{n^2} - \cos\frac{1}{(n+1)^2}\right)$ is convergent.

(7) Which one of the following series is NOT alternating series?

(A) $\sum_{n=1}^{\infty} (-1)^{n-1} \arctan n$ (B) $\sum_{n=1}^{\infty} \frac{n\cos(n\pi)}{2^n}$

(C) $\sum_{n=1}^{\infty} (-1)^n \sin\left(n\pi + \frac{\pi}{2}\right)$ (D) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n+4)}$

國立高雄大學理學院 108 學年度第 2 學期
微積分基礎能力會考試題

(8) Which one of the following statements is false?

(A) The series $\sum_{n=1}^{\infty}\left(1-\frac{1}{n}\right)^{n^2}$ is divergent.

(B) The series $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$ is divergent.

(C) The series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sqrt[4]{n}}$ is convergent.

(D) The series $\sum_{n=1}^{\infty}\frac{(2n)!}{(n!)^2}$ is divergent.

(9) Which one of the following statements is true?

(A) $\lim_{(x,y)\to(0,0)}\frac{x^4-4y^2}{x^2+2y^2}$ does not exist.

(B) $\lim_{(x,y)\to(0,0)}\frac{xy}{\sqrt{x^2+y^2}}$ does not exist.

(C) The function $f(x,y)=\arctan\left((x+y)^{-2}\right)$ is continuous on $\{(x,y)|x\neq y\}$.

(D) None of the above.

(10) Find the direction in which the function $f(x,y)=x^4y-x^2y^3$ decreases fastest at the point $(2,-3)$.

(A) $\langle 92,12\rangle$ (B) $\langle -92,-12\rangle$ (C) $\langle -12,92\rangle$ (D) $\langle 12,-92\rangle$

◎ 多選擇題 (多選五題，每題六分，共三十分。答錯一個選項扣三分，錯兩個選項以上不給分，分數不倒扣)

(1) Which of the following statements are true?

(A) $\int_{-1}^{1}\frac{1}{x^2}\,dx=-2$.

(B) $\int_{0}^{\infty}\left(x^2-\frac{1}{2}\right)e^{-x^2}\,dx=0$.

(C) $\int_{0}^{1}x^p\ln x\,dx$ converges if $p>-1$.

(D) $\int_{-\infty}^{\infty}x\,dx=0$.

(2) Consider the curve $x=\cos^3\theta$, $y=\sin^3\theta$, $0\le\theta\le\pi$. Which of the following statements are true?

(A) The area of the region enclosed by the curve and the x-axis is $\frac{3\pi}{16}$.

(B) The total length of the curve is 3.

(C) The area of the surface obtained by rotating the curve about the x-axis is $\frac{12\pi}{5}$.

(D) The curve is concave downward when $\theta=\frac{\pi}{4}$.

國立高雄大學理學院 108 學年度第 2 學期
微積分基礎能力會考試題

(3) Which of the following statements are true?

(A) The interval of convergence of the series $\sum_{n=2}^{\infty}(-1)^n\frac{x^n}{4^n\ln n}$ is $(-2,2]$.

(B) The interval of convergence of the series $\sum_{n=0}^{\infty}\frac{(-3)^n x^n}{\sqrt{n+1}}$ is $\left[-\frac{1}{3},\frac{1}{3}\right]$.

(C) The interval of convergence of the series $\sum_{n=0}^{\infty}\frac{n(x+2)^n}{3^{n+1}}$ is $(-5,1)$.

(D) The interval of convergence of the series $\sum_{n=0}^{\infty}n!\,x^n$ is $\{0\}$.

(4) Let

$$f(x,y)=\begin{cases}\dfrac{x^3y-xy^3}{x^2+y^2} & \text{if } (x,y)\neq(0,0),\\ 0 & \text{if } (x,y)=(0,0).\end{cases}$$

Which of the following statements are true?

(A) $f_x(0,0)=0$. (B) $f_y(0,0)=0$. (C) $f_{xy}(0,0)=1$. (D) $f_{yx}(0,0)=1$.

(5) Let $f(x,y)=3x-x^3-2y^2+y^4$. Which of the following statements are true?

(A) f has a local maximum at $(1,0)$. (B) f has a local maximum at $(1,1)$.

(C) f has a saddle point at $(-1,1)$. (D) f has a saddle point at $(1,-1)$.

◎ 填空題 (五題，每題六分，共三十分，答錯不倒扣)

(1) The interval of convergence of the Maclaurin series for the function $f(x)=\arctan x$ is $[-1,1]$.

(2) The tangent plane to the surface $\frac{x^2}{4}+y^2+\frac{z^2}{9}=3$ at the point $(-2,1,-3)$ is $-(x+2)+2(y-1)-\frac{2}{3}(z+3)=0$.

(3) Let R be the region in the xy-plane bounded by the graphs of $y=x^2$ and $y=1$. Then the absolute minimum value of $f(x,y)=2x-2xy+y^2$ on the region R is $-\frac{11}{16}$.

(4) The maximum value of the function $f(x,y)=xy$ on the ellipse $\frac{x^2}{8}+\frac{y^2}{2}=1$ is 2.

(5) $\int_0^2\int_{2x}^4\sin y^2\,dy\,dx=$ $-\frac{1}{4}\cos 16+\frac{1}{4}$.

國立高雄大學理學院 108 學年度第 2 學期
微積分基礎能力會考答案卷

系別：＿＿＿＿＿＿＿＿　　姓名：＿＿＿＿＿＿＿＿　　學號：＿＿＿＿＿＿＿＿

總分	初閱		複閱	

會考成績	

◎ 單選擇題 (單選十題，每題四分，共四十分，答錯不倒扣)

(1) C	(2) D	(3) A	(4) B	(5) D
(6) B	(7) C	(8) A	(9) A	(10) C

評分	初閱	
	複閱	

◎ 多選擇題 (多選五題，每題六分，共三十分。答錯一個選項扣三分，錯兩個選項以上不給分，分數不倒扣)

(1) BC	(2) ABC	(3) CD
(4) ABD	(5) AD	

評分	初閱	
	複閱	

◎ 填空題 (五題，每題六分，共三十分，答錯不倒扣)

(1) $[-1,1]$	(2) $-(x+2)+2(y-1)-\frac{2}{3}(z+3)=0$	(3) $-\frac{11}{16}$
(4) 2	(5) $-\frac{1}{4}\cos 16+\frac{1}{4}$	

評分	初閱	
	複閱	