

國立高雄大學理學院 110 學年度第 1 學期

微積分基礎能力會考試題

◎ 單選擇題 (單選十題，每題四分，共四十分，答錯不倒扣)

(1) If  $f$  is an even function and  $g$  is an odd function defined on  $(-\infty, \infty)$ , then which of the following statement must be true?

- (A)  $h(x) = g(|x|)$  is odd. (B)  $h(x) = |g(x)|$  is even.  
 (C)  $h(x) = f(x) \cdot g(x)$  is even. (D)  $h(x) = (f \circ g)(x)$  is odd.

(2) Find the value of  $\lim_{x \rightarrow \infty} (2^x + 3^x)^{\frac{1}{x}}$ .

- (A) 2; (B) 3; (C) 5; (D)  $\infty$ .

(3) Find the value of  $\lim_{x \rightarrow 1^+} [\ln(x) \cdot \ln(\ln(x))]$ .

- (A) 0; (B) 1; (C)  $e$ ; (D)  $\infty$ .

(4) Let  $f(x) = 2^x \arcsin\left(\frac{x}{2}\right)$ . Find the value of  $f'(1)$ .

- (A)  $\frac{\pi + 2\sqrt{3}}{3}$ ; (B)  $\frac{\pi + 4\sqrt{3}}{3}$ ; (C)  $\frac{\ln(2)\pi + 2\sqrt{3}}{3}$ ; (D)  $\frac{\ln(2)\pi + 4\sqrt{3}}{3}$ .

(5) Let

$$f(x) = x^2(x - 1)^3.$$

Which of the following statement is **false**?

- (A)  $f$  has exact 3 critical points.  
 (B)  $f$  has exact 1 local minimum value.  
 (C)  $f$  has exact 2 local maximum values.  
 (D)  $f$  has exact 3 inflection points.

(6) Find the global minimum value of  $f(x) = x^{4/3} - x - x^{1/3}$  on the interval  $[-1, 8]$ .

- (A) -2; (B) -1; (C) 0; (D) 3.

(7) Let  $f(x) = \frac{\ln(x^2)}{2x^2}$ . Which of the following statement is true?

- (A)  $f$  is increasing on  $(-1, 0)$ . (B)  $f$  is concave upward on  $(0, 1)$ .  
 (C)  $f$  has exact one inflection point. (D) The global maximum of  $f$  is  $\frac{1}{2e}$ .

(8) Define  $f(x) = \tan^2(x)$  on the interval  $[0, \frac{\pi}{2})$ . Find  $(f^{-1})'(1)$ .

- (A) -4; (B)  $-\frac{1}{4}$ ; (C)  $\frac{1}{4}$ ; (D) 4.

(9) Find the tangent line to the graph of  $\sin(x)y + \sin((x+1)^2y^2) = 0$  at the point  $(0, \sqrt{\pi})$ .

- (A)  $y = \left(\frac{1}{2} + \sqrt{\pi}\right)x + \sqrt{\pi}$ ; (B)  $y = \left(\frac{1}{2} - \sqrt{\pi}\right)x + \sqrt{\pi}$ ;  
 (C)  $y = \left(\frac{1}{2} + \sqrt{\pi}\right)x - \sqrt{\pi}$ ; (D)  $y = \left(\frac{1}{2} - \sqrt{\pi}\right)x - \sqrt{\pi}$ .

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(10) Find the value of  $\int_0^1 \frac{x^3}{(2x^2 + 3)^3} dx$ .

- (A)  $\frac{1}{300}$ ;                      (B)  $\frac{1}{125}$ ;                      (C)  $\frac{1}{16}$ ;                      (D)  $\frac{1}{8}$ .

◎ 多選擇題 (多選五題，每題六分，共三十分。答錯一個選項扣三分，錯兩個選項以上不給分，分數不倒扣)

(1) Define

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

Which of the following statements are true?

- (A)  $f$  is continuous on the interval  $[-1, 1]$ .  
 (B)  $f$  is differentiable on the interval  $[-1, 1]$ .  
 (C)  $f'$  is continuous on the interval  $[-1, 1]$ .  
 (D) The Mean Value Theorem can be applied to  $f$  on the interval  $[-1, 1]$ .

(2) Define

$$f(x) = \sqrt{x^2 + x} - x.$$

Which of the following statements are true?

- (A) The domain of  $f$  is  $(-\infty, -1] \cup [0, \infty)$ .  
 (B)  $f$  has two horizontal asymptotes.  
 (C)  $f$  has no vertical asymptote.  
 (D) The derivative of  $f$  from the right at  $x = 0$  exists.

(3) Let  $f$  be a function that is continuous on  $[a, b]$  and  $f''$  exists on  $(a, b)$ . Which of the following statements must be true?

- (A)  $f$  is differentiable on  $(a, b)$ .  
 (B) If the derivative of  $f$  from the right at  $x = a$  is positive and  $f''(x) > 0$  for all  $x$  in  $(a, b)$ , then  $f$  is increasing on  $[a, b]$ .  
 (C) If  $\frac{f(x) - f(a)}{x - a} > 0$  for all  $x$  in  $(a, b)$ , then  $f$  is increasing on  $[a, b]$ .  
 (D) If  $f$  is increasing on  $[a, b]$ , then  $f'(x) > 0$  for all  $x$  in  $(a, b)$ .

(4) Let  $f$  be a function whose second derivative  $f''$  exists on  $(-\infty, \infty)$ . Which of the following statements must be true?

- (A) If  $f$  satisfies  $\frac{d}{dx} \left( \int_0^x f(t) dt \right) = xe^{\sin(x)}$ , then  $f(x) = xe^{\sin(x)}$ .  
 (B) If  $f$  satisfies  $\int_0^x f'(t) dt = xe^{\sin(x)}$ , then  $f(x) = xe^{\sin(x)}$ .  
 (C)  $\frac{d}{dx} \left( \int_{-x}^x f(t) dt \right) = 0$ .

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(D)  $\frac{d}{dx} \left( \int_{-1}^1 f(t) dt \right) = 0.$

(5) Which of the following statements are true?

(A)  $\int_{-\pi/4}^{\pi/4} \frac{\sec(x)}{1 + \tan^2(x)} dx = 0.$

(B)  $\int_{-\pi/4}^{\pi/4} \frac{\sec^2(x)}{1 + \tan^2(x)} dx = \frac{\pi}{2}.$

(C)  $\int_{-\pi/4}^{\pi/4} \frac{\sec^4(x)}{1 + \tan^2(x)} dx = \frac{\pi^2}{4}.$

(D)  $\int \frac{1 + \tan^2(x)}{\sec^4(x)} dx = \frac{x + \sin(x) \cos(x)}{2} + C.$

◎ 填空题 (五題，每題六分，共三十分，答錯不倒扣)

(1) The value of  $\int_0^1 \frac{x+2}{\sqrt{4-x^2}} dx$  is  $\underline{2 - \sqrt{3} + \frac{\pi}{3}}$ .

(2) If  $f$  is continuous on  $(-\infty, \infty)$  and  $\int_1^{2x+3} \frac{f(t)}{1+t^2} dt = \arctan(x+1)$ , then  $f(5) = \underline{\frac{13}{5}}$ .

(3) Let  $f(x) = xe^x$ . Then the global maximum value of  $f(3\sin(x) + 4\cos(x))$  for  $x$  in  $(-\infty, \infty)$  is  $\underline{5e^5}$ .

(4) The value of  $\lim_{x \rightarrow 0^+} (\sin(x))^{\cos(x)-1}$  is  $\underline{1}$ .

(5) Find the value of  $k > 0$  such that two functions  $f(x) = x \ln(x)$  and  $g(x) = kx^2$  have exact one intersection point on  $(0, \infty)$ .  $k = \underline{e^{-1}}$ .

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微積分基礎能力會考答案卷

系別：\_\_\_\_\_ 姓名：\_\_\_\_\_ 學號：\_\_\_\_\_

總分	初閱		複閱	
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會考成績	
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◎ 單選擇題 (單選十題，每題四分，共四十分，答錯不倒扣)

(1) B	(2) B	(3) A	(4) C	(5) C
(6) B	(7) D	(8) C	(9) B	(10) A

評分	初閱	
	複閱	

◎ 多選擇題 (多選五題，每題六分，共三十分。答錯一個選項扣三分，錯兩個選項以上不給分，分數不倒扣)

(1) ABD	(2) AC	(3) AB
(4) AD	(5) BD	

評分	初閱	
	複閱	

◎ 填充題 (五題，每題六分，共三十分，答錯不倒扣)

(1) $2 - \sqrt{3} + \frac{\pi}{3}$	(2) $\frac{13}{5}$	(3) $5e^5$
(4) 1	(5) $e^{-1}$	

評分	初閱	
	複閱	