

國立高雄大學理學院 107 學年度第 2 學期
微積分基礎能力會考試題 (A 卷)

◎ 單選擇題 (單選十題，每題四分，共四十分，答錯不倒扣)

(1) Find the area of the region bounded by the graphs of $x = 3 - y^2$ and $x = y + 1$.

- (A) $\frac{16}{3}$. (B) $\frac{9}{2}$. (C) 4. (D) $\frac{10}{3}$.

(2) Find the volume of the solid formed by revolving the region bounded by $y = x^3$, $y = 1$, and $x = 0$ about the y -axis.

- (A) $\frac{9\pi}{14}$. (B) $\frac{9\pi}{7}$. (C) $\frac{3\pi}{5}$. (D) $\frac{3\pi}{10}$.

(3) Find the arc length of the graph of $y = \frac{e^x + e^{-x}}{2}$ on the interval $[0, 1]$.

- (A) $\frac{e - e^{-1}}{2}$. (B) $\frac{e + e^{-1}}{2} - 1$. (C) $\frac{e^2 - e^{-2}}{4}$. (D) $\frac{e^2 + e^{-2}}{4} - \frac{1}{2}$.

(4) Evaluate the definite integral $\int_0^1 \frac{x^3 e^{x^2}}{(x^2 + 1)^2} dx$.

- (A) $\frac{e - 1}{2}$. (B) $\frac{e - 2}{2}$. (C) $\frac{e - 1}{4}$. (D) $\frac{e - 2}{4}$.

(5) Evaluate the definite integral $\int_0^{\pi/4} \sec^3 x dx$.

- (A) $\sqrt{2} + \ln(1 + \sqrt{2})$. (B) $\sqrt{2} - \ln(1 + \sqrt{2})$.
(C) $\frac{\sqrt{2}}{2} + \frac{1}{2} \ln(1 + \sqrt{2})$. (D) $\frac{\sqrt{2}}{2} - \frac{1}{2} \ln(1 + \sqrt{2})$.

(6) Evaluate the definite integral $\int_0^{\pi/4} \sec^5 x \tan^3 x dx$.

- (A) $\frac{8\sqrt{2} - 1}{7} + \frac{4\sqrt{2} - 1}{5}$. (B) $\frac{8\sqrt{2} - 1}{7} - \frac{4\sqrt{2} - 1}{5}$.
(C) $\frac{8\sqrt{2} - 1}{7} + \frac{4\sqrt{2} + 1}{5}$. (D) $\frac{8\sqrt{2} - 1}{7} - \frac{4\sqrt{2} + 1}{5}$.

(7) Evaluate the definite integral $\int_{-3}^{-2} \frac{x}{\sqrt{x^2 + 6x + 10}} dx$.

- (A) $\sqrt{2} - 1 - 3 \ln(\sqrt{2} + 1)$. (B) $\sqrt{2} + 1 - 3 \ln(\sqrt{2} - 1)$.
(C) $\sqrt{2} - 1 + 3 \ln(\sqrt{2} + 1)$. (D) $\sqrt{2} - 1 - 3 \ln(\sqrt{2} - 1)$.

(8) Evaluate the definite integral $\int_0^{1/2} \frac{x}{x^4 - 1} dx$.

- (A) $\frac{1}{8} \ln \frac{5}{3}$. (B) $\frac{1}{8} \ln \frac{3}{5}$. (C) $\frac{1}{4} \ln \frac{5}{3}$. (D) $\frac{1}{4} \ln \frac{3}{5}$.

(9) Let $f(x) = x^3 + 3x - 1$. Find $(f^{-1})'(-5)$.

- (A) $-\frac{1}{6}$. (B) $\frac{1}{6}$. (C) $-\frac{1}{78}$. (D) $\frac{1}{78}$.

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(10) Which one of the following series converges?

(A) $\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^n$.

(B) $\sum_{n=1}^{\infty} \frac{n2^n}{4n^3 + 1}$.

(C) $\sum_{n=2}^{\infty} \ln \left(1 - \frac{1}{n^2}\right)$.

(D) $\sum_{n=1}^{\infty} \sin \frac{(2n-1)\pi}{2}$.

◎ 多選擇題 (多選四題，每題六分，共二十四分。答錯一個選項扣三分，錯兩個選項以上不給分，分數不倒扣)

(1) Which of the following statements are true?

(A) The range of $y = \arcsin x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

(B) The range of $y = \arccos x$ is $(0, \pi)$.

(C) The range of $y = \arctan x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \setminus \{0\}$.

(D) The range of $y = \operatorname{arcsec} x$ is $[0, \pi] \setminus \left\{\frac{\pi}{2}\right\}$.

(2) Which of the following statements are true?

(A) $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{e^x} = 2$.

(B) $\lim_{x \rightarrow 1^+} \left(\frac{1}{\ln x} - \frac{1}{x-1}\right) = \frac{1}{2}$.

(C) $\lim_{x \rightarrow 0^+} \frac{e^x - (1+x)}{x^3} = \infty$.

(D) $\lim_{x \rightarrow 0^+} \left(1 + \frac{1}{x}\right)^x = 1$.

(3) Which of the following statements are true?

(A) $\sum_{n=1}^{\infty} \left(\frac{-3n}{2n+1}\right)^{3n}$ is conditionally convergent.

(B) $\sum_{n=3}^{\infty} \frac{1}{n(\ln n)[\ln(\ln n)]^2}$ is absolutely convergent.

(C) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$ is conditionally convergent.

(D) $\sum_{n=1}^{\infty} \sin \frac{1}{n}$ is conditionally convergent.

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(4) Which of the following statements are true?

- (A) The interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n}$ is $(-1, 1]$.
- (B) The interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{(n+1)(n+2)}$ is $[-1, 1]$.
- (C) The interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n n!(x-5)^n}{3^n}$ is $\{5\}$.
- (D) The interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x-3)^{n+1}}{(n+1)4^{n+1}}$ is $[-1, 7)$.

◎ 填空題 (六題，每題六分，共三十六分，答錯不倒扣)

- (1) The volume of the solid formed by revolving the region bounded by the graphs of $y = x^3 + x + 1$, $y = 1$, and $x = 1$ about the line $x = 2$ is $\frac{29\pi}{15}$.
- (2) The area of the surface formed by revolving the graph of $f(x) = \frac{x^3}{6} + \frac{1}{2x}$ on the interval $[1, 2]$ about the x -axis is $\frac{47\pi}{16}$.
- (3) $\int_0^{2/3} \frac{6}{4+9x^2} dx = \frac{\pi}{4}$.
- (4) Let $f(x) = \arcsin x + \arccos x$. $f'(x) = 0$.
- (5) Let $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-2)^n}{n}$. The interval of convergence of $\int f(x) dx$ is $[1, 3]$.
- (6) Consider the sequence $\{a_n\}$, where $a_1 = 1$ and $a_{n+1} = \sqrt{1+a_n}$ for $n = 1, 2, 3, \dots$.
Then $\lim_{n \rightarrow \infty} a_n = \frac{1+\sqrt{5}}{2}$.

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系別：_____ 姓名：_____ 學號：_____

總分	初閱		複閱	
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會考成績	
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◎ 單選擇題 (單選十題，每題四分，共四十分，答錯不倒扣)

(1) B	(2) C	(3) A	(4) D	(5) C
(6) B	(7) A	(8) D	(9) B	(10) C

評分	初閱	
	複閱	

◎ 多選擇題 (多選四題，每題六分，共二十四分。答錯一個選項扣三分，錯兩個選項以上不給分，分數不倒扣)

(1) AD	(2) BCD	(3) BC
(4) ABCD		

評分	初閱	
	複閱	

◎ 填充題 (六題，每題六分，共三十六分，答錯不倒扣)

(1)	$\frac{29\pi}{15}$
(2)	$\frac{47\pi}{16}$
(3)	$\frac{\pi}{4}$
(4)	0
(5)	[1, 3]
(6)	$\frac{1 + \sqrt{5}}{2}$

評分	初閱	
	複閱	