# 一百一十一學年度第二學期微積分會考試題

## 一、單選擇題 (單選十題，每題五分，共五十分，答錯不倒扣)

1. The area of the region bounded by the curves $y = \ln(x)$, $y = 0$, and $x = e$ is

   (A) 1; (B) $e$; (C) ln2; (D) 2ln2.

2. Evaluate $\lim_{(x,y)\to(0,0)} \frac{sin(x^2+y^2)}{\sqrt{x^2+y^2}} =$

   (A) 0; (B) 1; (C) 0.5; (D) not exist.

3. Let $f(x,y) = x^2 + xy + y^2$ on the unit circle $x^2 + y^2 = 1$. Which one of the following statements is **True**?

   (A) $f$ has an absolute maximum at $(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$; (B) $f$ has an absolute minimum at (1, 0);

   (C) $f$ has one critical point; (D) The maximum value of $f$ is 1.5.

4. Let $f(x,y) = 3x^2y + y^3 - 6x^2 - 6y^2 + 2$. How many saddle points does $f$ possess?

   (A) 1; (B) 2; (C) 3; (D) 4.

5. Evaluate $\int_{-2}^{2} \frac{1}{x^3} =$

   (A) 0; (B) $\frac{3}{8}$; (C) $\frac{-3}{8}$; (D) not exist.

6. Evaluate $\int_{-7}^{7} \sqrt{49 - x^2}\, dx =$

   (A) $\frac{49\pi}{4}$; (B) $\frac{49\pi}{2}$; (C) $49\pi$; (D) 0.

7. Find the arc length of the curve $x(t) = a(t - sint), y(t) = a(1 - cost)$ on the interval $[0, 2\pi]$.

   (A) $2a$; (B) $4a$; (C) $6a$; (D) $8a$.

8. Evaluate $\int_{0}^{\pi/4} \tan^4\theta\, d\theta =$

   (A) $\frac{1+\pi}{4}$; (B) $\frac{\pi}{4} - \frac{2}{3}$; (C) $\frac{\pi}{4} - 1$; (D) $\frac{\pi}{4} + \frac{1}{3}$.

9. Let $w = 2xy$ with $x = s^2 + t^2$ and $y = s/t$. Find $\frac{\partial w}{\partial t} =$

   (A) $\frac{6s^2+2t^2}{t}$; (B) $\frac{s^2-2t^2}{st}$; (C) $\frac{2st^2-2s^3}{t^2}$; (D) $\frac{2ts^2+2st^2}{s}$.

10. Find the directional derivative of $h(x,y,z) = \ln(x + 2y + 2z)$ at $(1,1,1)$ in the direction from $(1,1,1)$ to $(1,5,4)$.

    (A) $\frac{3}{5}$; (B) $\frac{14}{5}$; (C) $\frac{14}{25}$; (D) $\frac{6}{25}$.

二、多選擇題 (多選五題，每題六分，共三十分。答錯一個選項扣三分，錯兩個選項以上不給分，分數不倒扣)

**11.** Consider the volumes of the solids generated by revolving the region bounded by $y = 0,\ x = 1, x = 5$ and $y = \frac{10}{x^2}$ about the given lines. Which of the following statements are **True** ?

(A) The $x$-axis. Then volume is $\frac{49}{15\pi}$;

(B) The $y$-axis. Then volume is $20\pi \ln(5)$ ;

(C) The line $y = -2$. Then volume is $\frac{1906}{15\pi}$;

(D) The line $y = 0$. Then volume is $\frac{496}{15\pi}$.

12. Let $f(x,y) = \begin{cases} \frac{x\sin(y)}{x^2+y^2} & if\ (x,y) \neq (0,0); \\ 0 & if\ (x,y) = (0,0). \end{cases}$ Which of the following statements are **True**?

(A) $f$ is continuous at $(0,0)$;

(B) $f_x(0,0) = 0$;

(C) $f_y(0,0) = 0$;

(D) $f$ is differentiable at $(0,0)$.

13. Let function $f(x,y) = \ln(\sqrt{x^2+y^2})$ and point $P = (2,1)$. Which of the following statements are **True** ?

(A) The gradient of $f$ at $P$ is $\langle \frac{1}{5}, \frac{2}{5} \rangle$ ;

(B) The directional derivative of $f$ at $P$ in the direction $\langle \frac{3}{5}, \frac{4}{5} \rangle$ is $\frac{2}{5}$;

(C) The minimum rate of change of $f$ at $P$ occurs in the direction $\frac{1}{\sqrt{5}}\langle 2,1 \rangle$ ;

(D) The maximum rate of change of $f$ at $P$ is $\frac{1}{\sqrt{5}}$.

14. For what values of $c$ does the integral $\int_0^\infty (\frac{1}{\sqrt{x^2+1}} - \frac{c}{x+1})dx$ converge?

(A) $ln(3)$; (B) $ln(2)$; (C) 1; (D) 2.

15. Which of the following statements are **Not True** ?

(A) If $f_x(x,y)$ and $f_y(x,y)$ are continuous, then $f(x,y)$ is differentiable;

(B) If $f(x,y)$ is differentiable, then $\nabla f(x,y)$ exists;

(C) If $\nabla f(a,b)$ exists, then $f(x,y)$ is continuous at $(a,b)$;

(D) If $f(x,y)$ is continuous at $(0,0)$, then $\nabla f(0,0)$ exists.

**三、填空題 (五題，每題四分，共二十分，答錯不倒扣)**

1. A right circular cone is generated by revolving the region bounded by $y = 0$, $x = 0$, and $y = 4 - \frac{4x}{3}$ about y-axis. The lateral surface area of the cone is ___(1)____.

2. Let $w(x, y, z)$ satisfy $\cos(xy) + \sin(yz) + zw = 20$. Then $\frac{\partial w}{\partial y} =$ ____(2)____.

3. For $u(t) = t^2\mathbf{i} - 2t\mathbf{j} + \mathbf{k}$. Then $\frac{d}{dt}[u(t) \times u'(t)]=$____(3)____.

4. The absolute maximum value of $f(x, y) = x^2 + 2y^2 - 2x + 3$ subject to the constraint $x^2 + y^2 \leq 10$ is ___(4)____.

5. $\int \frac{t^2-t-2}{t^3-2t-4} dt =$___(5)____.