國立高雄大學理學院 106 學年度第 1 學期
微積分基礎能力會考試題(A 卷)

◎ 單選擇題 (單選十題，每題四分，共四十分，答錯不倒扣)

1. Let $f(x) = \int_1^x \frac{\exp(-\sqrt{t})}{\sqrt{t}}\,dt$. Evaluate $\lim_{x\to\infty} f(x) =$

   (A) 0; (B) 1; (C) e^{-1}; (D) $2e^{-1}$.

2. Evaluate $\lim_{h\to 0}\frac{1-\cos h}{h^2}$

   (A) 0; (B) $\frac{1}{2}$; (C) 1; (D) $\frac{3}{2}$.

3. Let $f(x) = \frac{1}{8}x - 3$ and $g(x) = x^3$. Find $(f^{-1}\circ g^{-1})(1)$.

   (A) 33; (B) 32; (C) 31; (D) 30.

4. The integral $\int_{-1}^{1}\frac{x^2+\tan x}{1+x^2}\,dx =$

   (A) $2-\frac{\pi}{2}$; (B) $2-\frac{\pi}{4}$; (C) $1-\frac{\pi}{4}$; (D) $2-\pi$.

5. Consider $f(x) = 7x + \cos x$. Which of the following statements is **true** ?

   (A) $f(x)$ has infinitely many roots;
   (B) $f(x)$ is a periodic function;
   (C) $f(x)$ has exactly one root;
   (D) $f(x)$ has a slant asymptote $y = 7x$.

6. Evaluate $\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\frac{1}{1+(i/n)} =$

   (A) 0 ; (B) ln(2); (C) ln(3); (D) ∞.

7. Let $F(x) = \int_0^{x^3}\sin(t^2)\,dt$, find $F'(x) =$

   (A) $x^3\cos(x^6)$; (B) $x^3\sin(x^3)$; (C) $3x^2\cos(x^6)$; (D) $3x^2\sin(x^6)$.

8. On what interval is the curve $y = \int_0^x \frac{t^2}{t^2+t+2}\,dt$ concave downward ?

   (A) (0,4); (B) (4, ∞); (C) (-4,0); (D) (-∞,-4).

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9. Find the equation of the **tangent** line to the curve $y = x^{\cos(x)}$ at $x = \pi/2$.

(A) $y - 1 = -(x - \frac{\pi}{2})$; (B) $y - 1 = \ln(\pi/2)(x - \frac{\pi}{2})$;

(C) $y - 1 = -\ln\left(\frac{\pi}{2}\right)\left(x - \frac{\pi}{2}\right)$; (D) $y - 1 = \ln(\pi)(x - \frac{\pi}{2})$.

10. Find $\frac{dy}{dx}$, where $xe^y - xy + 3y^2 = 0$

(A) $\frac{y-e^y}{xe^y-x+6y}$; (B) $\frac{e^y-y}{xe^y-x+3y}$; (C) $\frac{x-e^y}{xe^y+x+3y}$; (D) $\frac{x+e^y}{xe^y+x+6y}$.

◎ 多選擇題 (多選五題，每題六分，共三十分。答錯一個選項扣三分，錯兩個選項以上不給分，分數不倒扣)

11. Which of the following statements are **not true** ?

(A) $\int_{-\pi/2}^{\pi/2} \sin^2(x)\cos(x)\, dx = 0$; (B) $\int_a^b f(x+h)\, dx = \int_{a+h}^{b+h} f(x)\, dx$;

(C) If $f(x) = x^n$, then f^{-1} exist; (D) $\ln(x^{1/2}) = \frac{1}{2}\ln(x)$.

12. Which of the following statements are **not true** ?

(A) If f is continuous on $[0,1]$, then $\int_0^1 f(x)dx = \int_0^1 f(1-x)dx$;

(B) If $2 \le f(x) \le 4$ for $0 \le x \le 1$, then $1 < \int_0^1 f(x)dx < 5$;

(C) If $f(x)$ is continuous on (a,b), then $f(x)$ is integrable on [a,b];

(D) $\frac{d}{dx}\left[\int_{u(x)}^{v(x)} f(t)dt\right] = f(v(x))v'(x) - f(u(x))u'(x)$.

13. Which of the following statements are **true** ?

(A) If f is continuous at a, so is $|f|$;

(B) If $x = a$ is a vertical asymptote of $y = f(x)$, then f may or may not be defined at a;

(C) If f is continuous at a, then f is differentiable at a;

(D) $\int_1^e \frac{\ln(x^2)}{x} dx = 2$.

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14. Suppose that $f'(a)=1$ for some constant a. The **possible values** for the limit

$$\lim_{x\to a}\frac{f(x)-f(a)}{\sqrt[3]{x}-1}$$ are

(A) 0; (B) 1; (C)2; (D) 3.

15. Consider

$$f(x)=\begin{cases} 0, & if\ x=0; \\ x\sin\left(\frac{1}{x}\right), & if\ x\neq 0. \end{cases}$$

Which of the following statements are **true**?

(A) $f(x)$ is differentiable at $x=0$;

(B) $f(x)$ is continuous at $x=0$;

(C) $f(x)$ has a global maximum value;

(D) $f(x)$ has an inflection point.

◎ 填空題 (五題，每題六分，共三十分，答錯不倒扣)

1. How many points of inflection does the function $f(x)=x^6-15x^2+1$ have ? ___2_____.

2. Solve the equation $\arccos(\sqrt{3x})=\arcsin(\sqrt{x})$. x=___1/4_____.

3. The slope of the tangent to $x^3+xy-\cos(xy)=0$ at the point (1,0) is ___-3_____.

4. Let $F(x)=\int_{x^3}^{1}\sqrt{14+2^t}\,dt$. Then the derivative $(F^{-1})'(0)$= ___-1/12_____.

5. The absolute maximum value of the function $f(x)=x\sqrt{9-x^2}$ with $-3\le x\le 3$ is ___9/2_____.