國立高雄大學理學院 107 學年度第 2 學期
微積分基礎能力會考試題 (B 卷)

◎ 單選擇題 (單選十題，每題四分，共四十分，答錯不倒扣)

(1) Find the area of the region bounded by the graphs of $x = 3 - y^2$ and $x = y + 1$.

(A) $\frac{16}{3}$. (B) $\frac{9}{2}$. (C) 4. (D) $\frac{10}{3}$.

(2) Find the volume of the solid formed by revolving the region bounded by $y = x^3$, $y = 1$, and $x = 0$ about the y-axis.

(A) $\frac{9\pi}{14}$. (B) $\frac{9\pi}{7}$. (C) $\frac{3\pi}{5}$. (D) $\frac{3\pi}{10}$.

(3) Find the arc length of the graph of $y = \frac{e^x + e^{-x}}{2}$ on the interval $[0, 1]$.

(A) $\frac{e - e^{-1}}{2}$. (B) $\frac{e + e^{-1}}{2} - 1$. (C) $\frac{e^2 - e^{-2}}{4}$. (D) $\frac{e^2 + e^{-2}}{4} - \frac{1}{2}$.

(4) Evaluate the definite integral $\int_0^1 \frac{x^3 e^{x^2}}{(x^2+1)^2}\, dx$.

(A) $\frac{e-1}{2}$. (B) $\frac{e-2}{2}$. (C) $\frac{e-1}{4}$. (D) $\frac{e-2}{4}$.

(5) Evaluate the definite integral $\int_0^{\pi/4} \sec^3 x\, dx$.

(A) $\sqrt{2} + \ln(1+\sqrt{2})$. (B) $\sqrt{2} - \ln(1+\sqrt{2})$.

(C) $\frac{\sqrt{2}}{2} + \frac{1}{2}\ln(1+\sqrt{2})$. (D) $\frac{\sqrt{2}}{2} - \frac{1}{2}\ln(1+\sqrt{2})$.

(6) Evaluate the definite integral $\int_0^{\pi/4} \sec^5 x \tan^3 x\, dx$.

(A) $\frac{8\sqrt{2}-1}{7} + \frac{4\sqrt{2}-1}{5}$. (B) $\frac{8\sqrt{2}-1}{7} - \frac{4\sqrt{2}-1}{5}$.

(C) $\frac{8\sqrt{2}-1}{7} + \frac{4\sqrt{2}+1}{5}$. (D) $\frac{8\sqrt{2}-1}{7} - \frac{4\sqrt{2}+1}{5}$.

(7) Evaluate the definite integral $\int_{-3}^{-2} \frac{x}{\sqrt{x^2+6x+10}}\, dx$.

(A) $\sqrt{2} - 1 - 3\ln(\sqrt{2}+1)$. (B) $\sqrt{2} + 1 - 3\ln(\sqrt{2}-1)$.

(C) $\sqrt{2} - 1 + 3\ln(\sqrt{2}+1)$. (D) $\sqrt{2} - 1 - 3\ln(\sqrt{2}-1)$.

(8) Evaluate the definite integral $\int_0^{1/2} \frac{x}{x^4 - 1}\, dx$.

(A) $\frac{1}{8}\ln\frac{5}{3}$. (B) $\frac{1}{8}\ln\frac{3}{5}$. (C) $\frac{1}{4}\ln\frac{5}{3}$. (D) $\frac{1}{4}\ln\frac{3}{5}$.

(9) For the curve given by $x = 2 + \sec\theta$ and $y = 1 + 2\tan\theta$, find $\frac{d^2y}{dx^2}$ when $\theta = -\frac{\pi}{3}$.

(A) $-\frac{16\sqrt{3}}{7}$. (B) $\frac{4}{3}$. (C) $-\frac{4\sqrt{3}}{3}$. (D) $\frac{2\sqrt{3}}{9}$.

國立高雄大學理學院 107 學年度第 2 學期
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(10) Which one of the following lines is a tangent line to the graph of $r = 2\cos 3\theta$ at the pole?

(A) $\theta = \frac{\pi}{6}$. (B) $\theta = \frac{\pi}{4}$. (C) $\theta = \frac{\pi}{3}$. (D) $\theta = 0$.

◎ 多選擇題 (多選四題，每題六分，共二十四分。答錯一個選項扣三分，錯兩個選項以上不給分，分數不倒扣)

(1) Let $\mathbf{u} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\mathbf{v} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$. Which of the following statements are true?

(A) $\mathbf{u} \cdot \mathbf{v} = -1$.

(B) $\mathbf{u} \times \mathbf{v} = 3\mathbf{i} + 5\mathbf{j} + 7\mathbf{k}$.

(C) $\text{proj}_{\mathbf{v}}\, \mathbf{u} = -\frac{1}{2}\mathbf{i} - \frac{1}{6}\mathbf{j} + \frac{1}{3}\mathbf{k}$.

(D) $\theta < \frac{\pi}{2}$, where θ is the angle between $\mathbf{u}$ and $\mathbf{v}$.

(2) Which of the following statements are true?

(A) The graph of the cylindrical equation $r^2\cos 2\theta + z^2 + 1 = 0$ is a hyperboloid of one sheet.

(B) The graph of the spherical equation $\phi = \frac{\pi}{2}$ is the xy-plane.

(C) The graph of the spherical equation $\rho = 4\csc\phi\sec\theta$ is a plane.

(D) The graph of the cylindrical equation $r = \pi$ is a cylinder.

(3) Let $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + \mathbf{k}$ and $\mathbf{s}(t) = (\cos t + t\sin t)\mathbf{i} + (\sin t - t\cos t)\mathbf{j} + t\mathbf{k}$. Which of the following statements are true?

(A) $\int_0^1 \mathbf{r}(t) \cdot \mathbf{s}(t)\, dt = 2$.

(B) $\mathbf{s}'(t)$ is perpendicular to $\mathbf{s}''(t)$ for all t.

(C) $\mathbf{r}(t)$ is perpendicular to $\mathbf{r}'(t)$ for all t.

(D) $\mathbf{s}'(0) \times \mathbf{s}''(0) = \mathbf{i}$.

(4) Which of the following statements are true?

(A) The direction cosines for the vector $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ are $\frac{2}{\sqrt{29}}$, $\frac{3}{\sqrt{29}}$, and $\frac{4}{\sqrt{29}}$.

(B) $\mathbf{j} \times \mathbf{k} = \mathbf{i}$.

(C) The area of the parallelogram that has the vectors $\mathbf{u} = -3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ and $\mathbf{v} = -2\mathbf{j} + 6\mathbf{k}$ as adjacent sides is $\sqrt{1036}$.

(D) The volume of the parallelepiped that has $\mathbf{u} = 3\mathbf{i} - 5\mathbf{j} + \mathbf{k}$, $\mathbf{v} = 2\mathbf{j} - 2\mathbf{k}$, and $\mathbf{w} = 3\mathbf{i} + \mathbf{j} + \mathbf{k}$ as adjacent edges is 36.

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微積分基礎能力會考試題 (B 卷)

◎ 填空題 (六題，每題六分，共三十六分，答錯不倒扣)

(1) Given the initial condition $y(0) = 2$, the particular solution of the equation $xy\,dx + e^{-x^2}(y^2 - 1)\,dy = 0$ is $y^2 - \ln y^2 + e^{x^2} = 5 - 2\ln 2$.

(2) The general solution of $(y-1)\sin x\,dx - dy = 0$, $x > 0$, is $y = 1 + Ce^{-\cos x}$.

(3) Consider the curve given by $x = \cos^3\theta$ and $y = \sin^3\theta$. Then $\dfrac{d^2y}{dx^2}$ when $\theta = \dfrac{\pi}{4}$ is $\dfrac{4\sqrt{2}}{3}$.

(4) The volume of the solid formed by revolving the region bounded by the graphs of $y = x^3 + x + 1$, $y = 1$, and $x = 1$ about the line $x = 2$ is $\dfrac{29\pi}{15}$.

(5) $\displaystyle\int_0^{\pi/2} [(5\cos t)\mathbf{i} + (6\sin t)\mathbf{j} + \mathbf{k}]\,dt =$ $5\mathbf{i} + 6\mathbf{j} + \dfrac{\pi}{2}\mathbf{k}$.

(6) A point is represented in cylindrical coordinates by $\left(6, \dfrac{\pi}{2}, -6\right)$. In spherical coordinates, the point is $\left(6\sqrt{2}, \dfrac{\pi}{2}, \dfrac{3\pi}{4}\right)$.

Integration Table ($a > 0$)

- $\displaystyle\int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln\left|u + \sqrt{u^2 \pm a^2}\right| + C.$

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系別：＿＿＿＿＿＿＿＿　　姓名：＿＿＿＿＿＿＿＿　　學號：＿＿＿＿＿＿＿＿

總分	初閱		複閱	

會考成績	

◎ 單選擇題 (單選十題，每題四分，共四十分，答錯不倒扣)

(1) B	(2) C	(3) A	(4) D	(5) C
(6) B	(7) A	(8) D	(9) D	(10) A

評分	初閱	
	複閱	

◎ 多選擇題 (多選四題，每題六分，共二十四分。答錯一個選項扣三分，錯兩個選項以上不給分，分數不倒扣)

(1) AB	(2) BCD	(3) C
(4) ABCD		

評分	初閱	
	複閱	

◎ 填空題 (六題，每題六分，共三十六分，答錯不倒扣)

(1) $y^2 - \ln y^2 + e^{x^2} = 5 - 2\ln 2$

(2) $y = 1 + Ce^{-\cos x}$

(3) $\dfrac{4\sqrt{2}}{3}$

(4) $\dfrac{29\pi}{15}$

(5) $5\mathbf{i} + 6\mathbf{j} + \dfrac{\pi}{2}\mathbf{k}$

(6) $\left(6\sqrt{2}, \dfrac{\pi}{2}, \dfrac{3\pi}{4}\right)$

評分	初閱	
	複閱	