一百一十三學年度第二學期微積分會考試題

◎ 單選擇題 (單選十題，每題五分，共五十分，答錯不倒扣)

(1) Find the area of the region bounded by the graphs of $f(x) = \sqrt[3]{x-1}$ and $g(x) = x - 1$.

(A) $\frac{1}{4}$. (B) $\frac{1}{2}$. (C) 1. (D) None of the above.

(2) Find the volume of the solid generated by revolving the region bounded by the graphs of $y = \sqrt{x}$, $y = -\frac{1}{2}x + 4$, $x = 0$, and $x = 8$ about the x-axis.

(A) $\frac{32}{3}\pi$. (B) $\frac{56}{3}\pi$. (C) 48π. (D) $\frac{88}{3}\pi$.

(3) $\displaystyle\int_3^5 \frac{2x}{x^2 - 4x + 4}\, dx = ?$

(A) $2\ln 3 + \frac{8}{3}$. (B) $2\ln 3 - \frac{8}{3}$. (C) $-2\ln 3 + \frac{16}{3}$. (D) $2\ln 3 + \frac{16}{3}$.

(4) $\displaystyle\int_0^\pi \sin^5 x\, dx = ?$

(A) $\frac{16}{15}$. (B) $\frac{8}{15}$. (C) $\frac{28}{15}$. (D) None of the above.

(5) $\displaystyle\int_0^2 \frac{3}{4x^2 + 5x + 1}\, dx = ?$

(A) $4\ln 3$. (B) $3\ln 3$. (C) $2\ln 3$. (D) $\ln 3$.

(6) Find the normal component of acceleration for space curve $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + \frac{t^2}{2}\mathbf{k}$ at $t = 1$.

(A) $\frac{\sqrt{6}}{6}$. (B) $\frac{\sqrt{30}}{6}$. (C) $\frac{5\sqrt{6}}{6}$. (D) None of the above.

(7) $\displaystyle\lim_{(x,y)\to(2,1)} \frac{x - y - 1}{\sqrt{x - y} - 1} = ?$

(A) 0. (B) 1. (C) 2. (D) does not exist.

(8) Find $\frac{\partial w}{\partial s}$ when $s = 1$ and $t = 2\pi$ for $w = xy + yz + xz$, where $x = s\cos t$, $y = s\sin t$, and $z = t$.

(A) 2π. (B) -2π. (C) $2 + 2\pi$. (D) $2 - 2\pi$.

(9) Find the directional derivative of $f(x, y, z) = xy + yz + xz$ at $(1, 2, -1)$ in the direction of $\mathbf{v} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$.

(A) $\frac{5\sqrt{6}}{6}$. (B) $-\frac{5\sqrt{6}}{6}$. (C) $\frac{\sqrt{6}}{6}$. (D) $-\frac{\sqrt{6}}{6}$.

(10) Which of the following points is a saddle point of $f(x, y) = 2xy - \frac{1}{2}(x^4 + y^4) + 1$?

(A) $(1, 1, 2)$. (B) $(-1, -1, 2)$. (C) $(0, 0, 1)$. (D) None of the above.

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◎ 多選擇題 (多選五題，每題六分，共三十分。答錯一個選項扣三分，錯兩個選項以上不給分，分數不倒扣)

(1) Consider the volumes of the solids generated by revolving the given region about the x-axis. Which of the following statements are **true**?

(A) If the region is bounded by $y = x^3$, $x = 0$, and $y = 8$, then the volume is $\frac{768\pi}{7}$.

(B) If the region is bounded by $x + y = 4$, $y = x$, and $y = 0$, then the volume is $\frac{16\pi}{3}$.

(C) If the region is bounded by $y = 3 - x$, $y = 0$, and $x = 6$, then the volume is 9π.

(D) If the region is bounded by $y = 1 - \sqrt{x}$, $y = x + 1$, and $y = 0$, then the volume is π.

(2) Which of the following statements are **true**?

(A) $\displaystyle\int_0^{\pi} x \sin 2x \, dx = -\frac{\pi}{2}$.

(B) $\displaystyle\int_0^{1} x \arcsin x^2 \, dx = \frac{\pi}{4} - 1$.

(C) $\displaystyle\int_0^{1} e^x \sin x \, dx = \frac{e(\sin 1 - \cos 1)}{2} - 1$.

(D) $\displaystyle\int_0^{\pi/8} x \sec^2 2x \, dx = \frac{\pi}{16} - \frac{1}{8}\ln 2$.

(3) Which of the following statements are **true**?

(A) $\displaystyle\int_0^{\sqrt{3}/2} \frac{1}{(1-x^2)^{5/2}} \, dx = \sqrt{3}$.

(B) $\displaystyle\int_0^{3/5} \sqrt{9 - 25x^2} \, dx = \frac{9\pi}{10}$.

(C) $\displaystyle\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1 + \sin^2 x} \, dx = \frac{\pi}{2}$.

(D) $\displaystyle\int_0^{\infty} \frac{dx}{\sqrt{x}(x+1)} = \pi$.

(4) Consider the function defined by

$$f(x,y) = \begin{cases} \dfrac{xy(x^2 - y^2)}{x^2 + y^2}, & (x,y) \neq (0,0), \\ 0, & (x,y) = (0,0). \end{cases}$$

Which of the following statements are **true**?

(A) $f_x(0,0) = 1$. (B) $f_y(0,0) = 0$. (C) $f_{xy}(0,0) = -1$. (D) $f_{yx}(0,0) = 1$.

(5) Consider the ellipsoid $S_1 : x^2 + 2y^2 + 2z^2 = 20$ and the paraboloid $S_2 : x^2 + y^2 + z = 4$. Which of the following statements are **true**?

(A) An equation of the tangent plane of S_1 at $(2,2,2)$ is $x + 2y + 2z = 10$.

(B) The angle of inclination of the tangent plane to S_1 at $(0,1,3)$ is $\arccos \frac{3}{\sqrt{10}}$.

(C) A set of symmetric equations for the normal line of S_2 at $(1,1,2)$ is $x - 1 = y - 1 = z - 2$.

(D) A set of parametric equations for the tangent line to the curve of intersection of S_1 and S_2 at $(0,1,3)$ is $x = t$, $y = 1$, and $z = 3$.

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◎ 填空題 (五題，每題四分，共二十分，答錯不倒扣)

(1) The arc length of the graph of $y = \frac{x^3}{6} + \frac{1}{2x}$ on the interval $[\frac{1}{2}, 2]$ is ________.

(2) Find $\displaystyle\int \frac{x}{(a+bx)^2}\,dx =$ ________.

(3) Let $\mathbf{r}$ be a differentiable vector-valued functions of t. Find $\frac{d}{dt}\|\mathbf{r}(t)\| =$ ________.

(4) The minimum value of $f(x, y) = 3x^2 + 2y^2 - 4y$ over the region in the xy-plane bounded by the graphs of $y = x^2$ and $y = 4$ is ________.

(5) Assume that x, y, and z are nonnegative. The minimum value of $f(x, y, z) = x^2 + y^2 + z^2$ subject to the constraints $x + 2z = 6$ and $x + y = 12$ is ________.