

國立高雄大學理學院 109 學年度第 1 學期
微積分基礎能力會考試題 (A 卷)

◎ 單選擇題 (單選十題，每題四分，共四十分，答錯不倒扣)

- (1) Let $f(x)$ be a function with domain \mathbb{R} . Which one of the following functions must be odd?
(A) $f(x) + f(-x)$. (B) $f(|x|)$. (C) $f(x) - f(-x)$. (D) $|f(x)|$.
- (2) The domain of $f(x) = \csc^{-1}(\ln x)$ is
(A) $[e^{-1}, e]$. (B) $(-\infty, e^{-1}] \cup [e, \infty)$.
(C) $(0, \infty)$. (D) $(-\infty, \infty)$.
- (3) $\lim_{x \rightarrow \infty} (x + 1 - \sqrt{x^2 + x}) = ?$
(A) $\frac{1}{2}$. (B) 1. (C) 0. (D) does not exist.
- (4) Which one of the following statements is **false**?
(A) If $f(x) < g(x)$ for all $x \neq a$, then $\lim_{x \rightarrow a} f(x) < \lim_{x \rightarrow a} g(x)$.
(B) $f(x) = \ln(x^2 + x + 1)$ is continuous on $(-\infty, \infty)$.
(C) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$.
(D) All of the above statements are true.
- (5) Find the derivative of $f(x) = \cos(5x^3 - 2)$.
(A) $15x^2 \cos(5x^3 - 2)$. (B) $-15x^2 \cos(5x^3 - 2)$.
(C) $15x^2 \sin(5x^3 - 2)$. (D) $-15x^2 \sin(5x^3 - 2)$.
- (6) Find the slope of the tangent line of the curve $x \sin(2y) = y \sin(2x)$ at the point $(\frac{\pi}{8}, \frac{\pi}{8})$.
(A) -1. (B) 0. (C) 1. (D) 2.
- (7) Let $f(x) = \ln \frac{(x+1)^8}{\sqrt{x-2}}$. $f'(3) = ?$
(A) $\frac{3}{2}$. (B) $-\frac{3}{2}$. (C) 3. (D) -3.
- (8) Let $a < c < b$ and let f be a function such that f'' exists and is continuous on $[a, b]$. Suppose that $f''(x) > 0$ for $x \in (a, b)$. Which one of the following statements is **false**?
(A) $f'(a) < \frac{f(b) - f(a)}{b - a}$. (B) $f'(b) > \frac{f(b) - f(a)}{b - a}$.
(C) $f(c) < f(a) + \frac{f(b) - f(a)}{b - a}(c - a)$. (D) $f(b) < f(c) + \frac{f(b) - f(a)}{b - a}(b - c)$.
- (9) Let $f(x) = \int_{\cos x}^{\sin^2 x} \ln(3 + 2v) dv$. Then $f'(\frac{\pi}{2}) = ?$
(A) 3. (B) $\ln 3$. (C) 1. (D) 0.

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(10) Evaluate the definite integral $\int_0^{\sin^{-1} \frac{8}{\sqrt{65}}} \frac{dx}{\cos^2 x \sqrt{1 + \tan x}}$.

- (A) 2. (B) 4. (C) 3. (D) 6.

◎ 多選擇題 (多選五題，每題六分，共三十分。答錯一個選項扣三分，錯兩個選項以上不給分，分數不倒扣)

(1) Define

$$f(x) = \begin{cases} 2 \sin(x) \cos(\frac{1}{x}) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

Which of the following statements are **true**?

- (A) The Intermediate Value Theorem can be applied to f on the interval $[-1, 1]$.
(B) The Mean Value Theorem can be applied to f on the interval $[-1, 1]$.
(C) f has a global maximum value.
(D) $\lim_{x \rightarrow \infty} f'(x)$ does not exist.

(2) Let $f(x) = \frac{x^2 + 1}{x^2 - 4}$. Which of the following statements are **true**?

- (A) f has no horizontal asymptote and no vertical asymptote.
(B) f has exact one critical point.
(C) f has exact one local maximum value but no local minimum value.
(D) f has exact one inflection point.

(3) Which of the following statements are **true**?

(A) $\int_{-2}^1 \frac{2}{x^3} dx = -\frac{3}{4}$. (B) $\int_{\pi/2}^{3\pi/2} \csc x \cot x dx = 2$.
(C) $\int_0^{\pi/4} \frac{1 - \sin^2 x}{\cos^2 x} dx = \frac{\pi}{4}$. (D) $\int_0^4 |x^2 - 4x + 3| dx = 4$.

(4) Which of the following statements are **true**?

(A) $\int_0^1 x^3(1-x)^5 dx = \int_0^1 x^5(1-x)^3 dx$.
(B) $\int_0^{\pi/2} \sin^3 x dx = \int_0^{\pi/2} \cos^3 x dx$.
(C) $\int_0^x f(t)(x-t) dt = \int_0^x \left(\int_0^t f(s) ds \right) dt$.
(D) $\int_0^1 \frac{\sin x}{\sin(1-x) + \sin x} dx = \frac{1}{2}$.

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(5) Which of the following statements are **true**?

(A) $\lim_{x \rightarrow 0} \frac{x^2}{x^2 + \sin x} = 1.$

(B) $\lim_{x \rightarrow 1} \frac{\ln x}{\sin \pi x} = -\frac{1}{\pi}.$

(C) $\lim_{x \rightarrow 0^+} \left(1 + \frac{1}{x}\right)^x = e.$

(D) $\lim_{x \rightarrow 0} \frac{\sin 5x}{\tan 9x} = \frac{5}{9}.$

◎ 填空題 (五題，每題六分，共三十分，答錯不倒扣)

(1) The absolute maximum of the function $f(x) = 2 \sin x - \cos 2x$ on the interval $[0, 2\pi]$ is 3.

(2) Use a linear approximation (or differentials) to estimate $\sqrt[3]{26}$. Then $\sqrt[3]{26} \approx \underline{3 - \frac{1}{27}}$.

(3) Find the area A of the largest rectangle that can be inscribed (內接) in a semicircle of radius r . Then $A = \underline{r^2}$.

(4) All points of inflection of the graph of the function $f(x) = \sin x + \cos x$ are $\left(\frac{3\pi}{4}, 0\right), \left(\frac{7\pi}{4}, 0\right)$.

(5) $\int_{-1}^1 \frac{\tan x}{1 + x^2 + x^4} dx = \underline{0}$.

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系別：_____ 姓名：_____ 學號：_____

總分	初閱		複閱	
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◎ 單選擇題 (單選十題，每題四分，共四十分，答錯不倒扣)

(1) C	(2) B	(3) A	(4) A	(5) D
(6) C	(7) A	(8) D	(9) B	(10) B

評分	初閱	
	複閱	

◎ 多選擇題 (多選五題，每題六分，共三十分。答錯一個選項扣三分，錯兩個選項以上不給分，分數不倒扣)

(1) ACD	(2) BC	(3) CD
(4) ABCD	(5) BD	

評分	初閱	
	複閱	

◎ 填充題 (五題，每題六分，共三十分，答錯不倒扣)

(1) 3	(2) $3 - \frac{1}{27}$	(3) r^2
(4) $(\frac{3\pi}{4}, 0), (\frac{7\pi}{4}, 0)$	(5) 0	

評分	初閱	
	複閱	