一百一十二學年度第二學期微積分會考試題

一、單選擇題 (單選十題，每題五分，共五十分，答錯不倒扣)

1. Find the area of the region between the graphs of $f(x) = 3x^3 - x^2 - 10x$, and $g(x) = 2x - x^2$.

   (A) 24; (B) 12; (C) 6; (D) 0.

2. Evaluate $\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2} =$

   (A) 0; (B) 1; (C) 0.5; (D) not exist.

3. Let $f(x,y) = -x^3 + 4xy - 2y^2 + 1$. Which one of the following statements is **True**?

   (A) f has a relative minimum at $(\frac{4}{3}, \frac{4}{3})$; (B) f has a relative maximum at (0, 0);

   (C) f has at least one saddle point; (D) f has a critical point at $(-1, \frac{4}{3})$.

4. Let $f(x,y) = (x^2 + 4y^2)e^{1-x^2-y^2}$. How many saddle points does f possess?

   (A) 4; (B) 3; (C) 2; (D) 1.

5. Evaluate $\int_{-1}^{1} \frac{1}{\sqrt{1-x^2}} dx =$

   (A) 0; (B) π; (C) $\frac{\pi}{2}$; (D) None of the above.

6. Evaluate $\lim_{x\to 0^+} (sinx)^x$ =

   (A) e^{-1}; (B) e; (C) 1; (D) 0.

7. Find the arc length of the space curve $r(t) = \langle\, 2sint, 5t, 2cost \,\rangle$on the interval $[0, \pi]$.

   (A) $\frac{33\pi}{4}$; (B) $\frac{65\pi}{4}$; (C) $\sqrt{29}$; (D) $\sqrt{29}\pi$.

8. Evaluate $\int_0^{\pi/4} 6\tan^3 x\, dx =$

   (A) $3(1 - ln2)$; (B) 3; (C) $\frac{\pi}{4} + 1$; (D) $\frac{\pi}{4} - 1$.

9. Let $w = 2xy$ with $x = s^2 + t^2$ and $y = s/t$. Find $\frac{\partial w}{\partial s}$ =

   (A) $\frac{6s^2+2t^2}{t}$; (B) $\frac{6s^2-2t^2}{t}$; (C) $\frac{2st^2-2s^3}{t^2}$; (D) $\frac{2st^2+2s^3}{t^2}$.

10. Find the directional derivative of $f(x,y) = x^2\sin(2y)$ at $(1, \pi/2)$ in the direction of $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$.

    (A) $\frac{3}{5}$; (B) $\frac{8}{5}$; (C) $\frac{14}{25}$; (D) $\frac{6}{25}$.

二、多選擇題 (多選五題，每題六分，共三十分。答錯一個選項扣三分，錯兩個選項以上不給分，分數不倒扣)

11. Consider the volumes of the solids generated by revolving the region bounded by $y = 0,\ x = 0, x = 2$ and $y = x^3$ about the given lines. Which of the following statements are **True** ?

(A) The x-axis. Then volume is $\frac{128\pi}{7}$;

(B) The y-axis. Then volume is $\frac{32\pi}{5}$

(C) The line $x = 3$. Then volume is $\frac{56\pi}{5}$;

(D) The y-axis. Then volume is $\frac{16\pi}{5}$.

12. Let $f(x,y) = \begin{cases} sin(xy)/(xy) & if\ xy \neq 0; \\ 1 & if\ xy = 0. \end{cases}$ Which of the following statements are **True**?

(A) f is not continuous at $(3,0)$;

(B) $f_x(1,0) = 0$;

(C) $f_y(0,2) = 0$;

(D) f has no absolute minimum.

13. Let function $f(x,y) = 3 - \frac{x}{3} - \frac{y}{2}$ and point $P = (3,2)$. Which of the following statements are **True** ?

(A) The gradient of f at P is $\langle \frac{-1}{3}, \frac{1}{2} \rangle$;

(B) The directional derivative of f at P in the direction $\langle -3, -4 \rangle$ is $\frac{3}{5}$;

(C) The minimum rate of change of f at P occurs in the direction $\frac{1}{\sqrt{13}}\langle -2, -3 \rangle$;

(D) The maximum rate of change of f at P is $\frac{\sqrt{13}}{6}$.

14. For what values of c does the integral $\int_e^{\infty} e^{clnx} dx$ converge?

(A) -4; (B) $ln(2)$; (C) 1; (D) -2.

15. Which of the following statements are **Not True** ?

(A) If $f(x,y)$ is differentiable, then $D_{\mathbf{u}} f(x,y) = \nabla f(x,y) \cdot \mathbf{u}$ for any unit vector $\mathbf{u}$;

(B) If $f(x,y)$ is differentiable, then $\nabla f(x,y)$ exists;

(C) If $\nabla f(a,b)$ exists, then $f(x,y)$ is continuous at (a,b);

(D) If $f(x,y)$ is continuous at $(0,0)$, then $\nabla f(0,0)$ exists.

三、填空題 (五題，每題四分，共二十分，答錯不倒扣)

1. Let $f(x,y)=\frac{xy}{\sqrt{x^2+y^2}}$. Then $\frac{3f_x(3,4)+4f_y(3,4)}{f(3,4)}=$ ____(1)____.

2. For $r(t)=2\cos(t)\mathbf{i}+3\sin(t)\mathbf{j}$. The maximum value of $\|r''(t)\|$ is ____(2)____.

3. The absolute maximum value of $f(x,y)=x^2+y^2-4xy+3$ subject to the constraint $x^2+y^2=1$ is ___(3)___.

4. Let $f(x)=\frac{8x^3+13x}{(x^2+2)^2}=\frac{Ax+B}{x^2+2}+\frac{Cx+D}{(x^2+2)^2}$. Then $A+B+C+D=$____(4)____. In addition, $\int_0^1 \frac{8x^3+13x}{(x^2+2)^2}dx=$___(5)___.