國立高雄大學理學院 114 學年度第 2 學期
微積分基礎能力會考試題

◎ 單選擇題 (單選十題，每題五分，共五十分，答錯不倒扣)

(1) Find the area of the region bounded by the graphs of $y = \sqrt{x}$, $y = -\sqrt{x}$, and $y = x - 2$.

(A) $\frac{7}{2}$. (B) 4. (C) 5. (D) $\frac{9}{2}$.

(2) Evaluate $\displaystyle\int_0^1 \frac{1}{(x^2+1)^{3/2}}\,dx$.

(A) $\frac{1}{2}$. (B) $\frac{\sqrt{2}}{2}$. (C) 1. (D) $\sqrt{2}$.

(3) Evaluate $\displaystyle\int_0^{\pi/2} x\cos^2 x\,dx$.

(A) $\frac{\pi^2}{16} - \frac{1}{4}$. (B) $\frac{\pi^2}{16}$. (C) $\frac{\pi^2}{8} - \frac{1}{4}$. (D) $\frac{\pi}{4} - \frac{1}{4}$.

(4) Find the arc length of the curve $\mathbf{r}(t) = t\,\mathbf{i} + \frac{t^2}{2}\,\mathbf{j} + \sqrt{2}\,t\,\mathbf{k}$ on the interval $0 \le t \le 1$.

(A) $1 + \frac{3}{4}\ln 3$. (B) $1 + \frac{3}{2}\ln 3$. (C) $2 + \frac{3}{4}\ln 3$. (D) $\frac{1}{2} + \frac{3}{4}\ln 3$.

(5) Find the volume of the solid generated by revolving the region under $y = \sqrt{x\ln x}$, above the x-axis, for $1 \le x \le e$ about the x-axis.

(A) $\frac{\pi}{4}(e^2-1)$. (B) $\frac{\pi}{2}(e^2-1)$. (C) $\frac{\pi}{4}(e^2+1)$. (D) $\frac{\pi}{2}(e^2+1)$.

(6) Let $z = f(u,v)$, where $u = x^2 + y$ and $v = xy$. Suppose $f_u(1,0) = 2$ and $f_v(1,0) = -1$. Find $\frac{\partial z}{\partial x}$ at $(x,y) = (1,0)$.

(A) 1. (B) 2. (C) 4. (D) -1.

(7) Find the complete range of p for which the improper integral $\displaystyle\int_2^\infty \frac{x^p}{x^4 - 2x^2 + 1}\,dx$ converges.

(A) $(-\infty, -1]$. (B) $(-\infty, -1)$. (C) $(-\infty, 3]$. (D) $(-\infty, 3)$.

(8) Evaluate $\displaystyle\lim_{(x,y)\to(0,0)} \frac{4x^2 + xy + y^2}{4x^2 + y^2}$.

(A) 1. (B) $\frac{6}{5}$. (C) $\frac{5}{4}$. (D) does not exist.

(9) Evaluate $\displaystyle\int_0^1 \frac{2x^2 + 7x + 7}{x^2 + 3x + 2}\,dx$.

(A) $2+\ln\dfrac{4}{3}$. (B) $2+3\ln 2-\ln 3$. (C) $2+\ln 3$. (D) $2+2\ln 2-\ln 3$.

(10) How many local minimum points does the function $f(x,y)=x^4+y^4-4xy$ have?

(A) 1. (B) 2. (C) 3. (D) 4.

◎ 多選擇題 (多選五題，每題六分，共三十分。答錯一個選項扣三分，錯兩個選項以上不給分，分數不倒扣)

(11) Consider the region R bounded by $y=e^x$, $y=1$, and $x=1$. Which of the following expressions for the volume of the solid generated by revolving R are true?

(A) About the x-axis: $V=\pi\displaystyle\int_0^1 (e^{2x}-1)\,dx$.

(B) About the line $x=2$: $V=2\pi\displaystyle\int_0^1 (2-x)e^x\,dx$.

(C) About the y-axis: $V=2\pi\displaystyle\int_0^1 x(e^x-1)\,dx$.

(D) About the line $y=1$: $V=\pi\displaystyle\int_0^1 (e^{2x}-1)\,dx$.

(12) Which of the following integration statements are true?

(A) $\displaystyle\int_0^1 \frac{\ln x}{\sqrt{x}}\,dx=-2$. (B) $\displaystyle\int_0^{1/2} \arcsin x\,dx=\frac{\pi}{12}+\frac{\sqrt{3}}{2}$.

(C) $\displaystyle\int_0^{\pi/4} \tan^2 x\,dx=1-\frac{\pi}{4}$. (D) $\displaystyle\int_1^2 \frac{1}{x^2\sqrt{x^2+4}}\,dx=\frac{\sqrt{5}-\sqrt{2}}{4}$.

(13) Which of the following statements are true?

(A) If f_x and f_y exist in a neighborhood of (a,b), then f is continuous at (a,b).

(B) If f_x and f_y exist in a neighborhood of (a,b) and are continuous at (a,b), then f is differentiable at (a,b).

(C) If $\nabla f(a,b)=\mathbf{0}$, $f_{xx}(a,b)>0$, and $f_{yy}(a,b)>0$, then f has a local minimum at (a,b).

(D) If f is differentiable at (a,b), then for every unit vector $\mathbf{u}$, the directional derivative $D_{\mathbf{u}}f(a,b)$ exists and $D_{\mathbf{u}}f(a,b)=\nabla f(a,b)\cdot\mathbf{u}$.

(14) Consider the function $f(x,y)=\begin{cases}\frac{x^3y}{x^4+y^2}, & (x,y)\neq(0,0),\\ 0, & (x,y)=(0,0).\end{cases}$ Which of the following statements are true?

(A) f is continuous at $(0,0)$.

(B) $f_x(0,0) = 0$ and $f_y(0,0) = 0$.

(C) f is differentiable at $(0,0)$.

(D) For every unit vector $\mathbf{u}$, the directional derivative $D_{\mathbf{u}}f(0,0)$ exists and equals 0.

(15) Let S be the level surface $F(x,y,z) = x^2+2y^2+3z^2 = 12$, and let $P = (1,2,1)$. Which of the following statements are true?

(A) The vector $\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$ is tangent to S at P.

(B) An equation of the tangent plane to S at P is $x + 4y + 3z = 12$.

(C) A set of parametric equations for the normal line to S at P is $x = 1 + t,\ y = 2 + 2t,\ z = 1 + 3t$.

(D) There exists a differentiable curve $\mathbf{r}(t)$ on S with $\mathbf{r}(0) = P$ such that $\mathbf{r}'(0) = \mathbf{i} - \mathbf{j} + \mathbf{k}$.

◎ 填空題 (五題，每題四分，共二十分，答錯不倒扣)

(1) Find the absolute maximum value of $f(x,y) = x^2y$ on the region $x^2 - 2xy + 3y^2 \leq 3$. _____.

(2) Find the principal unit normal vector for the curve $\mathbf{r}(t) = t\,\mathbf{i} + t^2\,\mathbf{j} + \dfrac{2}{3}t^3\,\mathbf{k}$ at $t = 1$.

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(3) Find parametric equations for the tangent line to the curve of intersection of surfaces $z = x^2 + 2y^2$ and $z = 3x + y^2 - 1$ at the point $(1,1,3)$. Then $(x,y,z) =$ (_______, ______, _______) .

(4) Find the unit vector in which $f(x,y,z) = x^2e^{yz} + yz^2 + z$ decreases most rapidly at the point $(1,0,2)$. __________________________.

(5) Find the surface area generated by revolving the line segment $y = 3 - \frac{3}{4}x$ for $0 \leq x \leq 4$ about the x-axis. _______.