一百零六學年度第二學期微積分會考試題(A 卷)

一、單選擇題 (單選十題，每題五分，共五十分，答錯不倒扣)

1. Which one of the following series is divergent ?

   (A) $\sum_{n=1}^{\infty}\frac{1}{n^{1.5}}$; (B) $\sum_{n=1}^{\infty}\frac{n^3}{3^n}$; (C) $\sum_{n=0}^{\infty}\frac{(-1)^n\sqrt{n}}{n+1}$; (D) $\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}}-\frac{1}{n}\right)$

2. Evaluate $\lim_{(x,y)\to(0,0)}\frac{sin(y)-y}{\sqrt{x^2+y^2}}$

   (A) 0; (B) 1; (C) $\frac{1}{6}$; (D) not exist.

3. Let $f(x,y)=x^2+xy+y^2$ on the unit circle $x^2+y^2=1$. Which one of the following statements is **true** ?

   (A) f has an absolute maximum at $(\frac{-1}{\sqrt{2}},\frac{1}{\sqrt{2}})$. (B) f has an absolute minimum at (1, 0).

   (C) f has one critical point. (D) The maximum value of f is 1.5

4. Let $f(x,y)=3x^2y+y^3-6x^2-6y^2+2$. How many saddle points does f possess?

   (A) 1 (B) 2; (C) 3; (D) 4;

5. If the integral $\int_0^{\infty}(\frac{x}{x^2+1}-\frac{a}{3x+1})dx$ is convergent, then $a=$.

   (A) 0; (B) 1; (C) 2; (D) 3;

6. Evaluate $\lim_{x\to 0^+}x^{\sqrt{x}}$

   (A) e^{-1}; (B) e; (C) 1; (D) 0.

7. The interval of convergence for $\sum_{n=0}^{\infty}\frac{(-3)^n x^n}{\sqrt{n+1}}$ is

   (A) $[\frac{-1}{3},\frac{1}{3}]$; (B) $(\frac{-1}{3},\frac{1}{3}]$; (C) $(\frac{-1}{3},\frac{1}{3})$; (D) $[\frac{-1}{3},\frac{1}{3})$.

8. Evaluate $\int_0^{\pi/4}\tan^3\theta\sec^2\theta\,d\theta$

   (A) 1 ; (B) $\frac{1}{2}$; (C) $\frac{1}{4}$; (D) $\frac{1}{8}$;

9. Let $w=x^2+y^2+z^2$, $x=tsin(s), y=tcos(s), z=st^2$. Find $\frac{\partial w}{\partial t}=$

   (A) $3\cos(s)+s^2t^3$; (B) $2s+s^2t^3$; (C) $3t+st^2$; (D) $4s^2t^3+2t$;

10. Find the directional derivative of $h(x,y,z)=\ln(3x+6y+9z)$ at (1,1,1) in the direction from (1,1,1) to (5,13,7).

    (A) $\frac{23}{42}$; (B) $\frac{17}{42}$; (C) $\frac{8}{21}$; (D) $\frac{9}{21}$.

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二、多選擇題 (多選五題，每題六分，共三十分。答錯一個選項扣三分，錯兩個選項以上不給分，分數不倒扣)

11. Which of the following statements are **Not True** ?

(A) If $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ is convergent.

(B) If $0 \le a_n \le b_n$ and $\sum b_n$ converges, then $\sum a_n$ converges.

(C) If $0 < a_n$ and $\sum a_n$ converges, then $\sum(-1)^n a_n$ converges.

(D) If $\{a_n\}$ and $\{b_n\}$ are divergent, then $\{a_n b_n\}$ is divergent

12. Let $f(x,y) = \begin{cases} \frac{x^2y+yx^2}{x^3+y^3} & if\ (x,y) \neq (0,0) \\ 0 & if\ (x,y) = (0,0) \end{cases}$

Which of the followings statements are **True**?

(A) f is continuous at $(0,0)$;

(B) $f_x(0,0) = 0$;

(C) $f_y(0,0) = 0$;

(D) f is differentiable at $(0,0)$;

13. Let function $f(x,y) = \ln(\sqrt[3]{x^2+y^2})$ and point $P = (1,2)$.

Which of the following statements are **True** ?

(A) The gradient of f at P is $\langle \frac{2}{15}, \frac{5}{15} \rangle$.

(B) The directional derivative of f at P in the direction $\langle \frac{3}{5}, \frac{4}{5} \rangle$ is $\frac{22}{75}$.

(C) The minimum rate of change of f at P occurs in the direction $\frac{1}{\sqrt{5}}\langle 1,2 \rangle$.

(D) The maximum rate of change of f at P is $\frac{2\sqrt{5}}{15}$.

14. Which of the following values of x the series $\sum_{n=0}^{\infty} \frac{nx^n}{(n+1)(2x+1)^n}$ converges absolutely ?

(A) $x = -1$; (B) $x = -2$; (C) $x = 1$; (D) $x = -0.25$.

15. Which of the following statements are **Not True** ?

(A) If f_x and f_y are continuous, then $f(x,y)$ is differentiable.

(B) If $f(x,y)$ is differentiable, then ∇f exists;

(C) If ∇f exists, then $f(x,y)$ is continuous;

(D) If $f(x,y)$ is continuous, then $f_x(0,0) = \lim_{(x,y)\to(0,0)} f_x(x,y)$

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三、填空題 (五題，每題四分，共二十分，答錯不倒扣)

1. Let $f(x) = \cos(x^2)$, then $f^{(12)}(0) =$ ____$-(12!)/6!$____

2. Let $w(x, y, z)$ satisfy $\cos(xy) + \sin(yz) + zw = 1$. Then $\left.\frac{\partial w}{\partial z}\right|_{(x,y,z)=(1,1,1)} =$ ____sin(1)-1___

3. $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots$ is ____ln(2)____

4. $\int \left(sec^2 t + \frac{1}{1+t^2}\right) dt =$____Tan(t)+arctan(t)+C____.

5. The absolute maximum value of $f(x, y, z) = xy - yz$ subject to the constraint $x^2 + y^2 + \frac{z^2}{2} = 6$ is ____$3\sqrt{3}$____