一百一十三學年度第一學期微積分會考試題

單選擇題(單選十題,每題五分,共五十分,答錯不倒扣) 0

(1) Let f(x) be a function with domain \mathbb{R} . Which one of the following functions must be even?

(A)
$$|f(x)|$$
. (B) $f(|x|)$. (C) $f(x) - f(-x)$. (D) None of the above.

(2) The range of $y = \operatorname{arccsc} x$ is

(A)
$$(0,\pi)$$
. (B) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. (C) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right] \setminus \{0\}$. (D) $[0,\pi] \setminus \{\frac{\pi}{2}\}$.
(3) $\lim_{x \to -\infty} (x + \sqrt{x^2 + 2x}) = ?$

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$$\lim_{x \to -\infty} (x + \sqrt{x^2 + 2x}) =$$

(A) -1. (B) 0. (C) 1. (D) does not exist.

- (4) Which one of the following statements is **true**?
 - (A) If f is continuous at a, so is |f|. (B) $f(x) = \ln(x^2)$ is continuous on $(-\infty, \infty)$.
 - (C) $\lim_{x \to \infty} \frac{\sin x}{x} = 1.$
 - (D) None of the above.
- (5) Find the derivative of $f(x) = \ln(\sin x)$.
 - (A) $\csc x$. (B) $\tan x$. (C) $\sin x$. (D) None of the above.

(6) Find the slope of the tangent line of the curve $e^y \cos x = 1 + \sin(xy)$ at the point (0, 0).

(A) -1. (B) 0. (C) 1. (D) None of the above.
(7) Let
$$f(x) = \ln \frac{x(x^2+1)^2}{\sqrt{2x^3-1}}$$
. $f'(1) =$?
(A) 2. (B) 1. (C) 0. (D) None of the above.

- (8) Let f be differentiable on (0, 4). Suppose that the graph of f is concave downward on (0, 4). Which one of the following statements must be **true**?
 - (A) f'(1) > f'(2). (B) f(1) > f(2). (C) f'' < 0 on (0, 4). (D) None of the above.

(9) Let
$$f(x) = \int_{x}^{2x} \frac{\sin t}{t} dt$$
. Then $f'(\frac{\pi}{2}) =$?
(A) $-\frac{\pi}{2}$. (B) $\frac{2}{\pi}$. (C) 0. (D) None of the above.
(10) $\lim_{h \to 0} \frac{\cos(\pi + h) - 2\cos(\pi) + \cos(\pi - h)}{h^2} =$?.
(A) -1. (B) 0. (C) 1. (D) does not exist.

第1頁/共3頁

一百一十三學年度第一學期微積分會考試題

- 多選擇題(多選五題,每題六分,共三十分。答錯一個選項扣三分,錯兩個選項以上不給分,分數不倒扣)
 - (1) Let

$$f(x) = \begin{cases} \sin(x^2)\sin(\frac{1}{x^2}) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

Which of the following statements are **true**?

- (A) The Intermediate Value Theorem can be applied to f on the interval [-1, 1].
- (B) The Mean Value Theorem can be applied to f on the interval [-1, 1].
- (C) f has a global maximum value.
- (D) f' is continuous at 0.

(2) Let $f(x) = x^{2/3}(6-x)^{1/3}$. Which of the following statements are **true**?

- (A) f has no horizontal asymptote and no vertical asymptote.
- (B) f has exact one critical point.
- (C) f has exact one local maximum value but no local minimum value.
- (D) f has exact one inflection point.
- (3) Which of the following statements are **true**?

(A)
$$\int_{\pi/4}^{3\pi/4} \sec^2 x \, dx = -2.$$
 (B) $\int_{\pi/2}^{3\pi/2} \csc^2 x \cot x \, dx = 0.$
(C) $\int_0^{\pi/4} \frac{\sec^2 x}{\tan^2 x + 1} \, dx = \frac{\pi}{4}.$ (D) $\int_0^2 |x^2 - 3x + 2| \, dx = 1.$

(4) Which of the following statements are **true**?

(A)
$$\int_{1}^{4} \frac{dx}{\sqrt{x}(2\sqrt{x}+5)} = 2\ln 3.$$
 (B) $\int_{0}^{\pi/2} \sin^{5} x dx = \int_{0}^{\pi/2} \cos^{5} x dx.$
(C) $\int_{0}^{\pi/2} 2^{\sin x} \cos x dx = \frac{1}{\ln 2}.$ (D) $\int_{-1}^{1} |x| \arctan x dx = 0.$

(5) Which of the following statements are **true**?

(A)
$$\lim_{x \to 0} \frac{x}{x + \arcsin x} = \frac{1}{2}$$
.
(B) $\int_{2}^{\sqrt{3}+2} \frac{dx}{x^2 - 4x + 7} = \frac{\pi}{4\sqrt{3}}$.
(C) $\lim_{x \to 0^+} \left(1 + \frac{1}{x}\right)^x = e$.
(D) $\lim_{x \to \infty} \frac{x}{x + \sin x} = 1$.

第2頁/共3頁

一百一十三學年度第一學期微積分會考試題

◎ 填空題 (五題,每題四分,共二十分,答錯不倒扣)

- (1) The absolute maximum of the function $f(x) = \sin^2 x + \cos x$ on the interval $[0, 2\pi]$ is
- (2) Write sec[arcsin(x-1)] in algebraic form: _____.
- (3) Evaluate the limit $\lim_{x \to 0} \frac{e^{-1/x^2}}{x^2} = \underline{\qquad}$.
- (4) All points of inflection of the graph of the function $f(x) = \frac{\cos x}{2 + \sin x}$, $x \in [0, 2\pi]$, are
- (5) Evaluate the integral $\int_0^1 \frac{dx}{2\sqrt{3-x}\sqrt{x+1}} =$ _____.