國立高雄大學理學院 114 學年度第 1 學期
微積分基礎能力會考試題

◎ 單選擇題 (單選十題，每題五分，共五十分，答錯不倒扣)

(1) Let $f(x) = \dfrac{x^2 + ax + b}{x - 1}$. If the limit $\lim_{x \to 1} f(x)$ exists and is equal to 3, find the value of $a - b$.

(A) 1. (B) 2. (C) 3. (D) 4.

(2) Evaluate $\lim_{x \to \infty} \left(\sqrt{x^2 + 4x} - x \right)$.

(A) 0. (B) 2. (C) 4. (D) ∞.

(3) Let $y = x^{\sqrt{x}}$ for $x > 0$. Find $\frac{dy}{dx}$.

(A) $\sqrt{x} \cdot x^{\sqrt{x}-1}$. (B) $x^{\sqrt{x}} \dfrac{\ln x + 2}{2\sqrt{x}}$.

(C) $x^{\sqrt{x}} \dfrac{\ln x}{2\sqrt{x}}$. (D) $x^{\sqrt{x}}(\ln x + 1)$.

(4) Let $f(x) = \arcsin(2x)$. Find $f'(\frac{1}{4})$.

(A) $\dfrac{2}{\sqrt{3}}$. (B) $\dfrac{4}{\sqrt{3}}$. (C) $\dfrac{1}{\sqrt{3}}$. (D) $\dfrac{8}{\sqrt{3}}$.

(5) Find the slope of the tangent line to the curve $x^2 + \sin y = xy$ at the point $(1, \pi/2)$.

(A) $2 - \pi/2$. (B) $\frac{2-\pi/2}{\pi/2}$. (C) 2. (D) Does not exist.

(6) Which of the following functions satisfies all hypotheses of the Mean Value Theorem on the interval $[-1, 1]$?

(A) $f(x) = |x|$. (B) $f(x) = x^{2/3}$. (C) $f(x) = \frac{1}{x}$. (D) $f(x) = \dfrac{1}{x-2}$.

(7) Find the x-coordinate where the function $f(x) = xe^{-2x}$ has a local maximum.

(A) $x = 0$. (B) $x = 1/2$. (C) $x = 1$. (D) $x = 2$.

(8) Evaluate $\lim_{n \to \infty} \sum_{i=1}^{n} \dfrac{3}{n} \sqrt{1 + \dfrac{3i}{n}}$.

(A) 7. (B) 14/3. (C) 16/3. (D) 8.

(9) Let $F(x) = \displaystyle\int_2^{x^2} \ln(t)\, dt$. Find $F'(e)$.

(A) 1. (B) $2e$. (C) $4e$. (D) $2e \ln(2)$.

(10) Evaluate $\displaystyle\int_0^1 \frac{e^x}{(1+e^x)^2}\,dx$.

(A) $\dfrac{1}{2}-\dfrac{1}{1+e}$. (B) $1-\dfrac{1}{1+e}$.

(C) $\ln(1+e)-\ln 2$. (D) $\dfrac{e}{(1+e)^2}$.

◎ 多選擇題 (多選五題，每題六分，共三十分。答錯一個選項扣三分，錯兩個選項以上不給分，分數不倒扣)

(11) Define $f(x)=\begin{cases} x^2\sin(\frac{1}{x}) & \text{if } x\neq 0 \\ 0 & \text{if } x=0 \end{cases}$. Which of the following statements are true?

(A) $\lim_{x\to 0} f(x)=0$.

(B) f is continuous at $x=0$.

(C) f is NOT differentiable at $x=0$.

(D) $f'(x)$ is continuous at $x=0$.

(12) Let $f(x)=\dfrac{e^{2x}-1}{x^2-x}$. Which of the following statements are true?

(A) The graph of f has a vertical asymptote at $x=0$.

(B) $\lim_{x\to 0} f(x)=-2$.

(C) The graph of f has a vertical asymptote at $x=1$.

(D) The graph of f has a horizontal asymptote at $y=0$.

(13) Let $f(x)=x(|x|-3)^{2/3}$. Which of the following statements are true?

(A) f is an odd function.

(B) f has a local maximum at $x=\frac{9}{5}$.

(C) $x=0$ is a critical point of f.

(D) f has a local minimum at $x=3$.

(14) Which of the following statements are true?

(A) If f is continuous on the open interval (a,b), then f attains an absolute maximum value on (a,b).

(B) If f is differentiable at c, then f must be continuous at c.

(C) If $f''(c)=0$, then $(c,f(c))$ is an inflection point of the curve $y=f(x)$.

(D) If $F(x)=\displaystyle\int_0^x f(t)\,dt$, then F is differentiable on its domain.

(15) Which of the following statements involving integrals are true?

(A) $\displaystyle\int_{-1}^{1} \frac{\sin(x^3)}{1+x^2}\,dx = 0.$

(B) $\displaystyle\int_{0}^{\pi} \tan(x)\,dx = 0.$

(C) $\displaystyle\int_{-1}^{0} \frac{1}{x^2+2x+2}\,dx = \frac{\pi}{4}.$

(D) $\displaystyle\int_{0}^{1} \frac{1}{\sqrt{4-x^2}}\,dx = \frac{\pi}{3}.$

◎ 填空題 (五題，每題四分，共二十分，答錯不倒扣)

(1) Evaluate the limit: $\displaystyle\lim_{x\to\infty}\left(\frac{x+2}{x-1}\right)^{2x} =$ ______.

(2) Let $f(x) = x^5 - 5x^4 + x + 8$. Find $(f^{-1})'(5)$. _________.

(3) Find the absolute maximum value of the function $f(x) = x\arcsin x + \sqrt{1-x^2}$ on the interval $[-1,1]$. _______.

(4) Evaluate the definite integral: $\displaystyle\int_{0}^{\ln 2} \frac{e^x}{e^{2x}+4e^x+5}\,dx =$ ____________________.

(5) Evaluate the limit: $\displaystyle\lim_{x\to 0}\frac{1}{x^3}\int_{x}^{2x}\sin(t^2)\,dt =$ _______.