國立高雄大學理學院 106 學年度第 1 學期
微積分基礎能力會考試題(B 卷)

◎ **單選擇題** **(單選二十五題，每題四分，共一百分，答錯不倒扣)**

1. Let $f(x)=\int_1^x \frac{\exp(-\sqrt{t})}{\sqrt{t}}dt$. Evaluate $\lim_{x\to\infty} f(x)=$

(A) 0; (B) 1; (C) e^{-1}; (D) $2e^{-1}$.

2. Evaluate $\lim_{h\to 0}\frac{1-\cos h}{h^2}$

(A) 0; (B) $\frac{1}{2}$; (C) 1; (D) $\frac{3}{2}$.

3. Let $f(x)=\frac{1}{8}x-3$ and $g(x)=x^3$. Find $(f^{-1}\circ g^{-1})(1)$.

(A) 33; (B) 32; (C) 31; (D) 30.

4. The integral $\int_{-1}^{1}\frac{\tan x}{1+x^2}\,dx$ =

(A) 0; (B) 1 (C) 2; (D) ∞.

5. Consider $f(x)=7x+\cos x$. Which of the following statements is **true** ?

(A) $f(x)$ has infinitely many roots;

(B) $f(x)$ is a periodic function;

(C) $f(x)$ has exactly one root;

(D) $f(x)$ has a slant asymptote $y=7x$.

6. Evaluate $\int_0^{\pi} sec^2(x/4)\,dx$ =

(A) 8 ; (B) 4; (C) 2; (D) 1.

7. Let $F(x)=\int_0^{x^3}\sin(t^2)\,dt$, find $F'(x)=$

(A) $x^3\cos(x^6)$; (B) $x^3\sin(x^3)$; (C) $3x^2\cos(x^6)$; (D) $3x^2\sin(x^6)$.

8. On what interval is the curve $y=\int_0^x \frac{t^2}{t^2+t+2}dt$ concave downward ?

(A) (0,4); (B) (4, ∞); (C) (-4,0); (D) (-∞,-4).

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9. Find the equation of the **tangent** line to the curve $y = x^{\cos(x)}$ at $x = \pi/2$.

(A) $y - 1 = -(x - \frac{\pi}{2})$; (B) $y - 1 = \ln(\pi/2)(x - \frac{\pi}{2})$;

(C) $y - 1 = -\ln\left(\frac{\pi}{2}\right)\left(x - \frac{\pi}{2}\right)$; (D) $y - 1 = \ln(\pi)(x - \frac{\pi}{2})$.

10. Find $\frac{dy}{dx}$, where $xe^y - xy + 3y^2 = 0$

(A) $\frac{y-e^y}{xe^y-x+6y}$; (B) $\frac{e^y-y}{xe^y-x+3y}$; (C) $\frac{x-e^y}{xe^y+x+3y}$; (D) $\frac{x+e^y}{xe^y+x+6y}$.

11. Which of the following statements are **not true** ?

(A) $\int_{-\pi/2}^{\pi/2} \sin^2(x)\cos(x)\,dx = \frac{2}{3}$; (B) $\int_a^b f(x+h)\,dx = \int_{a+h}^{b+h} f(x)\,dx$;

(C) If $f(x) = x^n$, then f^{-1} exist; (D) $\ln\left(x^{1/2}\right) = \frac{1}{2}\ln(x)$.

12. Which of the following statements are **not true** ?

(A) If f is continuous on [0,1], then $\int_0^1 f(x)dx = \int_0^1 f(1-x)dx$;

(B) If $2 \le f(x) \le 4$ for $0 \le x \le 1$, then $1 < \int_0^1 f(x)dx < 5$;

(C) If $f(x)$ is continuous on [a,b], then $f(x)$ is integrable on [a,b];

(D) $\frac{d}{dx}\left[\int_{u(x)}^{v(x)} f(t)dt\right] = f\big(v(x)\big)v'(x) - f\big(u(x)\big)u'(x)$.

13. Which of the following statements are **true** ?

(A) $\int_1^e \frac{\ln(x^2)}{x}dx = 2$;

(B) If $x = a$ is a vertical asymptote of $y = f(x)$, then $f(a)$ exists;

(C) If f is continuous at a, then f is differentiable at a;

(D) If f is continuous at a, so is $|f|$.

14. How many points of inflection does the function $f(x) = x^6 - 15x^2 + 1$ have ?

(A) 0; (B) 1; (C) 2; (D) 3.

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15. How many real roots does the equation $4x^5 + x^3 + 2x + 4 = 0$ have ?
(A) 1; (B) 2; (C) 3; (D) 5.

16. The slope of the tangent to $x^3 + xy - \cos(xy) = 0$ at the point (1,0) is
(A) 0; (B) -1; (C) -2; (D) -3.

17. Let $F(x) = \int_{x^3}^{1} \sqrt{14 + 2^t}\,dt$. Then the derivative $(F^{-1})'(0)$ =
(A) $\frac{-1}{12}$; (B) $\frac{1}{12}$; (C) $\frac{-1}{24}$; (D) $\frac{1}{24}$.

18. The absolute maximum value of the function $f(x) = x\sqrt{9 - x^2}$, $-3 \le x \le 3$ is
(A) $\frac{1}{2}$; (B) 0; (C) $\frac{9}{2}$; (D) $\frac{\sqrt{3}}{2}$.

19. Suppose that $f'(a) = 1$ for some constant a. The **possible value "a"** for the limit $\lim_{x\to a} \frac{f(x)-f(a)}{\sqrt[3]{x}-1} = 3$ is
(A) 0; (B) 1; (C)2; (D) 3.

20. Let f be a continuous function on $[1,4]$ and $\int_1^4 f(x)\,dx = 10$.
Which of the following statements is **always true** ?
(A) The average value of f on the interval $[1,4]$ is equal to 3;
(B) The maximum value of f on the interval $[1,4]$ is less than 4;
(C) The minimum value of f on the interval $[1,4]$ is greater than 0;
(D) The maximum value of f on the interval $[1,4]$ is greater than 3.

21. Which of the following statements are **not true** ?
(A) $\lim_{x\to\pi} \frac{\sin x}{x} = 0$; (B) $\lim_{x\to\infty} \frac{\sin}{x} = 1$;
(C) $\lim_{x\to 0} \frac{\sin x}{x} = 1$; (D) $\lim_{x\to -\infty} \frac{\sin x}{x} = 0$;

22. For what values of a does the curve $y = ax^3 + e^x$ have no inflection point ?
(A) -1; (B) 0; (C) 1; (D) 2.

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23. Given that $g(3)=3, g'(3)=7, h(6)=3, h'(6)=-2$, and let $f(x)=\frac{g(h(x))}{h(x)}$.

Then $f'(6)=$

(A) $\frac{-8}{3}$; (B) $\frac{-16}{3}$; (C) -2; (D) -4.

24. Let $f(x)=\frac{e^x-e^{-x}}{2}$. Then $f^{-1}(x)=$

(A) $\ln(x+\sqrt{x^2-1})$; (B) $\ln(x-\sqrt{x^2-1})$;
(C) $\ln(\sqrt{x^2+1}+x)$; (D) $\ln(\sqrt{x^2+1}-x)$.

25. Consider

$$f(x)=\begin{cases}1, & if\ x=0;\\ xsin\left(\frac{1}{x}\right), & if\ x\neq 0.\end{cases}$$

Which of the following statements are **true**?

(A) $f(x)$ is differentiable at $x=0$;
(B) $f(x)$ is continuous at $x=0$;
(C) $f(x)$ has a global maximum value;
(D) $f(x)$ has an inflection point.