

國立高雄大學理學院 108 學年度第 1 學期  
微積分基礎能力會考試題 (B 卷)

◎ 單選擇題 (單選二十題，每題四分，共八十分，答錯不倒扣)

(1) Which one of the following functions is even?

(A)  $f_1(x) = x^3 - x$ .

(B)  $f_2(x) = -x^2 - x - 1$ .

(C)  $f_3(x) = |x - 3|$ .

(D)  $f_4(x) = \frac{1}{x^2}$ .

(2) Which one of the following equations defines  $y$  as a function of  $x$ ?

(A)  $x^2y - x^2 + 4y = 0$ .

(B)  $x^2 + y^2 = 16$ .

(C)  $y^2 = x - 1$ .

(D)  $xy^2 = 1$ .

(3)  $\lim_{x \rightarrow 0} \frac{|x|}{x} = ?$

(A) 1.

(B) 0.

(C) -1.

(D) does not exist.

(4)  $\lim_{x \rightarrow 0^-} \frac{|x|^3}{x^3} = ?$

(A) 1.

(B) 0.

(C) -1.

(D) does not exist.

(5) Which one of the following statements is false?

(A) The graph of  $xy = 4$  is symmetric with respect to the origin.

(B) The graph of  $y = 2x^3 - x$  is symmetric with respect to the  $y$ -axis.

(C) The graph of  $|y| - x = 3$  is symmetric with respect to the  $x$ -axis.

(D) The graph of  $y = \frac{x}{x^2 + 1}$  is symmetric with respect to the origin.

(6) Let  $f(x) = \frac{\sin x}{x}$ . Find  $f'\left(\frac{\pi}{6}\right)$ .

(A)  $\frac{3\sqrt{3} + 18}{\pi^2}$ .

(B)  $\frac{3\sqrt{3} - 18}{\pi^2}$ .

(C)  $\frac{3 + 18\sqrt{3}}{\pi^2}$ .

(D)  $\frac{3 - 18\sqrt{3}}{\pi^2}$ .

(7) Let  $f(x) = \left(\frac{x+5}{x^2+2}\right)^2$ . Find  $f'(1)$ .

(A) -2.

(B) 2.

(C) -4.

(D) 4.

(8) Determine the slope of the graph of  $3(x^2 + y^2)^2 = 100xy$  at the point  $(3, 1)$ .

(A)  $\frac{13}{9}$ .

(B)  $-\frac{13}{9}$ .

(C)  $\frac{9}{13}$ .

(D)  $-\frac{9}{13}$ .

(9) Consider the function  $f(x) = (x^2 - 4)^{2/3}$ . Which one of the following statements is false?

(A)  $f$  has a relative minimum at the point  $(-2, 0)$ .

(B)  $f$  has a relative maximum at the point  $(0, \sqrt[3]{16})$ .

(C)  $f$  is increasing on the interval  $-2 < x < 0$ .

(D)  $f$  is decreasing on the interval  $2 < x < \infty$ .

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- (10) Consider the function  $f(x) = \frac{x}{\sqrt{x^2 + 2}}$ . Which one of the following statements is false?
- (A)  $f$  has an absolute maximum on  $(-\infty, \infty)$ .  
(B) The graph of  $f$  has at least one point of inflection.  
(C) The graph of  $f$  has at least one horizontal asymptote.  
(D) The graph of  $f$  is concave downward on the interval  $0 < x < \infty$ .
- (11) Evaluate the definite integral  $\int_0^2 |2x - 1| dx$ .
- (A)  $\frac{3}{2}$ . (B)  $\frac{5}{2}$ . (C) 2. (D) 3.
- (12) Define  $f(x) = \int_0^{x^3} \sin t^2 dt$ . Find  $f'(1)$ .
- (A)  $3 \cos 1$ . (B)  $\cos 1$ . (C)  $3 \sin 1$ . (D)  $\sin 1$ .
- (13) Evaluate the definite integral  $\int_{\pi/8}^{\pi/4} (\csc 2x - \cot 2x) dx$ .
- (A)  $\frac{1}{4} \ln \left(1 + \frac{\sqrt{2}}{2}\right)$ . (B)  $\frac{1}{4} \ln \left(1 - \frac{\sqrt{2}}{2}\right)$ . (C)  $\frac{1}{2} \ln \left(1 + \frac{\sqrt{2}}{2}\right)$ . (D)  $\frac{1}{2} \ln \left(1 - \frac{\sqrt{2}}{2}\right)$ .
- (14) Which one of the following statements is false?
- (A) If  $f$  is continuous on its domain, then  $f^{-1}$  is continuous on its domain.  
(B) If  $f$  is increasing on its domain, then  $f^{-1}$  is increasing on its domain.  
(C) If  $f$  is differentiable on an interval containing  $c$ , then  $f^{-1}$  is differentiable at  $f(c)$ .  
(D) All the above statements are true.
- (15) Evaluate the definite integral  $\int_0^1 (5^x - 3^x) dx$ .
- (A)  $\frac{4}{\ln 5} - \frac{2}{\ln 3}$ . (B)  $\frac{4}{\ln 5} + \frac{2}{\ln 3}$ . (C)  $\frac{2}{\ln 5} - \frac{4}{\ln 3}$ . (D)  $\frac{2}{\ln 5} + \frac{4}{\ln 3}$ .
- (16) Evaluate the limit  $\lim_{x \rightarrow 0^+} (e^x + x)^{2/x}$ .
- (A) 1. (B)  $e^4$ . (C)  $e^3$ . (D)  $e^2$ .
- (17) Evaluate  $\csc \left[ \arctan \left( -\frac{5}{12} \right) \right]$ .
- (A)  $\frac{13}{5}$ . (B)  $-\frac{13}{5}$ . (C)  $\frac{13}{12}$ . (D)  $-\frac{13}{12}$ .

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(18) Assume that  $f$  is continuous everywhere and that  $c$  is a constant. Then  $\int_{ca}^{cb} f(x) dx = ?$

- (A)  $\frac{1}{c} \int_a^b f(x) dx.$     (B)  $\frac{1}{c} \int_a^b f(cx) dx.$     (C)  $c \int_a^b f(x) dx.$     (D)  $c \int_a^b f(cx) dx.$

(19) Which one of the following lines is the slant asymptote of the graph of  $f(x) = \frac{2x^2 - 4x}{x + 1}$ ?

- (A)  $y = 2x - 12.$     (B)  $y = 2x.$     (C)  $y = 2x - 4.$     (D)  $y = 2x - 6.$

(20) Which one of the following statements is true?

- (A) If  $f$  is continuous at  $a$ , so is  $|f|$ .  
 (B) If  $|f|$  is continuous at  $a$ , so is  $f$ .  
 (C) If  $f(x) > 1$  for all  $x$  and  $\lim_{x \rightarrow 0} f(x)$  exists, then  $\lim_{x \rightarrow 0} f(x) > 1$ .  
 (D) none of the above.

◎ 多選擇題 (多選五題，每題四分，共二十分。答錯一個選項扣兩分，錯兩個選項以上不給分，分數不倒扣)

(1) Define

$$f(x) = \begin{cases} x^3 \sin \frac{1}{x} & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

Which of the following statements are true?

- (A)  $f$  is continuous on  $(-\infty, \infty)$ .    (B)  $f$  is differentiable on  $(-\infty, \infty)$ .  
 (C)  $f'$  is continuous on  $(-\infty, \infty)$ .    (D)  $f'$  is differentiable on  $(-\infty, \infty)$ .

(2) Let  $f(x) = x \sin x$ . Which of the following statements are true?

- (A)  $f^{(16)}(0) = 0.$     (B)  $f^{(16)}\left(\frac{\pi}{6}\right) = -8 + \frac{\sqrt{3}\pi}{12}.$   
 (C)  $f^{(16)}\left(\frac{\pi}{4}\right) = -8\sqrt{2} + \frac{\sqrt{2}\pi}{8}.$     (D)  $f^{(16)}\left(\frac{\pi}{2}\right) = \frac{\pi}{2}.$

(3) Which of the following statements are true?

- (A) If  $3 \leq f'(x) \leq 5$  for all values of  $x$ , then  $18 \leq f(8) - f(2) \leq 30$ .  
 (B)  $|\sin x| \leq |x|$  for  $x \in (-\infty, \infty)$ .  
 (C)  $|\cos x| \leq |x|$  for  $x \in (-\infty, \infty)$ .  
 (D)  $|\cos a - \cos b| \leq |a - b|$  for all  $a$  and  $b$ .

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(4) Which of the following statements are true?

(A)  $\int_{-2}^1 \frac{1}{x^4} dx = -\frac{3}{8}$ .

(B)  $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[ 4 - \left( 1 + \frac{i}{n} \right)^2 \right] \left( \frac{1}{n} \right) = \frac{5}{3}$ .

(C) If  $f$  is continuous on  $[a, b]$ , then  $\frac{d}{dx} \left( \int_a^b f(x) dx \right) = f(x)$ .

(D) If  $f'$  is continuous on  $[1, 3]$ , then  $\int_1^3 f'(x) dx = f(3) - f(1)$ .

(5) Which of the following statements are true?

(A) If a line contains points in both the first and third quadrants, then its slope must be positive.

(B) If  $(-4, -5)$  is a point on a graph that is symmetric with respect to the  $x$ -axis, then  $(4, -5)$  is also a point on the graph.

(C) The function  $y = 3 \cos \left( \frac{x}{3} \right)$  has a period that is three times that of the function  $y = \cos x$ .

(D)  $\sinh^2 x + \cosh^2 x = 1$ .

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系別：\_\_\_\_\_ 姓名：\_\_\_\_\_ 學號：\_\_\_\_\_

總分	初閱		複閱	
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會考成績	
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◎ 單選擇題 (單選二十題，每題四分，共八十分，答錯不倒扣)

(1) <b>D</b>	(2) <b>A</b>	(3) <b>D</b>	(4) <b>C</b>	(5) <b>B</b>
(6) <b>B</b>	(7) <b>C</b>	(8) <b>A</b>	(9) <b>D</b>	(10) <b>A</b>
(11) <b>B</b>	(12) <b>C</b>	(13) <b>C</b>	(14) <b>C</b>	(15) <b>A</b>
(16) <b>B</b>	(17) <b>B</b>	(18) <b>D</b>	(19) <b>D</b>	(20) <b>A</b>

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(1) <b>ABC</b>	(2) <b>CD</b>	(3) <b>ABD</b>
(4) <b>BD</b>	(5) <b>AC</b>	

評分	初閱	
	複閱	