

國立高雄大學理學院 109 學年度第 1 學期  
微積分基礎能力會考試題 (B 卷)

◎ 單選擇題 (單選十題，每題四分，共四十分，答錯不倒扣)

- (1) Let  $f(x)$  be a function with domain  $\mathbb{R}$ . Which one of the following functions must be odd?
- (A)  $f(x) + f(-x)$ . (B)  $f(|x|)$ . (C)  $f(x) - f(-x)$ . (D)  $|f(x)|$ .
- (2) Let  $f(x) = \frac{x}{x^2 + 1}$ . Which one of the following statements is **true**?
- (A) The graph of  $f$  is symmetric with respect to the line  $y = x$ .  
(B) **The graph of  $f$  is symmetric with respect to the origin.**  
(C) The graph of  $f$  is symmetric with respect to the  $x$ -axis.  
(D) The graph of  $f$  is symmetric with respect to the  $y$ -axis.
- (3)  $\lim_{x \rightarrow \infty} (x + 1 - \sqrt{x^2 + x}) = ?$
- (A)  $\frac{1}{2}$ . (B) 1. (C) 0. (D) does not exist.
- (4) Which one of the following statements is **false**?
- (A) **If  $f(x) < g(x)$  for all  $x \neq a$ , then  $\lim_{x \rightarrow a} f(x) < \lim_{x \rightarrow a} g(x)$ .**  
(B)  $f(x) = \ln(x^2 + x + 1)$  is continuous on  $(-\infty, \infty)$ .  
(C)  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$ .  
(D) All of the above statements are true.
- (5) Find the derivative of  $f(x) = \cos(5x^3 - 2)$ .
- (A)  $15x^2 \cos(5x^3 - 2)$ . (B)  $-15x^2 \cos(5x^3 - 2)$ .  
(C)  $15x^2 \sin(5x^3 - 2)$ . (D)  **$-15x^2 \sin(5x^3 - 2)$ .**
- (6) Find the slope of the tangent line of the curve  $x \sin(2y) = y \sin(2x)$  at the point  $(\frac{\pi}{8}, \frac{\pi}{8})$ .
- (A) -1. (B) 0. (C) **1**. (D) 2.
- (7) Let  $f(x) = \ln \frac{(x+1)^8}{\sqrt{x-2}}$ .  $f'(3) = ?$
- (A)  $\frac{3}{2}$ . (B)  $-\frac{3}{2}$ . (C) 3. (D) -3.
- (8)  $\int_0^3 \frac{x^2 + x + 1}{x^2 + 1} dx = ?$
- (A)  $3 - \ln 10$ . (B)  $3 + \ln 10$ . (C)  $3 - \frac{1}{2} \ln 10$ . (D)  **$3 + \frac{1}{2} \ln 10$ .**
- (9) Let  $f(x) = \sin x$ ,  $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ . Then  $(f^{-1})'(\frac{1}{2}) = ?$
- (A)  $-\frac{2}{\sqrt{3}}$ . (B)  $\frac{2}{\sqrt{3}}$ . (C) -2. (D) 2.

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(10) Evaluate the definite integral  $\int_0^{\arcsin \frac{8}{\sqrt{65}}} \frac{dx}{\cos^2 x \sqrt{1 + \tan x}}$ .

- (A) 2.                                      (B) 4.                                      (C) 3.                                      (D) 6.

◎ 多選擇題 (多選五題，每題六分，共三十分。答錯一個選項扣三分，錯兩個選項以上不給分，分數不倒扣)

(1) Which of the following statements are **true**?

- (A)  $\operatorname{arccsc}(-\sqrt{2}) = \arcsin\left(-\frac{1}{\sqrt{2}}\right)$ .                      (B)  $\arcsin^2\left(\frac{1}{2}\right) + \arccos^2\left(\frac{1}{2}\right) = 1$ .  
 (C)  $\frac{d}{dx}[\operatorname{arcsec} x] = \frac{1}{|x|\sqrt{x^2-1}}$ .                      (D)  $\sinh^2 3 = \frac{-1 + \cosh 6}{2}$ .

(2) Which of the following statements are **true**?

- (A)  $\int_0^2 \frac{x}{x^4 + 25} dx = \frac{1}{5} \arctan\left(\frac{4}{5}\right)$ .  
 (B)  $\int_0^{1/6} \frac{3}{\sqrt{1-9x^2}} dx = \frac{\pi}{6}$ .  
 (C)  $\int_1^4 \frac{1}{x\sqrt{16x^2-5}} dx = \frac{1}{\sqrt{5}} \operatorname{arcsec}\left(\frac{16}{\sqrt{5}}\right) - \frac{1}{\sqrt{5}} \operatorname{arcsec}\left(\frac{4}{\sqrt{5}}\right)$ .  
 (D)  $\int_{\pi/2}^{\pi} \frac{\sin x}{1 + \cos^2 x} dx = \frac{\pi}{2}$ .

(3) Which of the following statements are **true**?

- (A)  $\int_{-2}^1 \frac{2}{x^3} dx = -\frac{3}{4}$ .                                      (B)  $\int_{\pi/2}^{3\pi/2} \csc x \cot x dx = 2$ .  
 (C)  $\int_0^{\pi/4} \frac{1 - \sin^2 x}{\cos^2 x} dx = \frac{\pi}{4}$ .                                      (D)  $\int_0^4 |x^2 - 4x + 3| dx = 4$ .

(4) Which of the following statements are **true**?

- (A)  $\int_0^1 x^3(1-x)^5 dx = \int_0^1 x^5(1-x)^3 dx$ .  
 (B)  $\int_0^{\pi/2} \sin^3 x dx = \int_0^{\pi/2} \cos^3 x dx$ .  
 (C)  $\int_0^x f(t)(x-t) dt = \int_0^x \left( \int_0^t f(s) ds \right) dt$ .  
 (D)  $\int_0^1 \frac{\sin x}{\sin(1-x) + \sin x} dx = \frac{1}{2}$ .

(5) Which of the following statements are **true**?

- (A)  $\lim_{x \rightarrow 0} \frac{x^2}{x^2 + \sin x} = 1$ .                                      (B)  $\lim_{x \rightarrow 1} \frac{\ln x}{\sin \pi x} = -\frac{1}{\pi}$ .  
 (C)  $\lim_{x \rightarrow 0^+} \left(1 + \frac{1}{x}\right)^x = e$ .                                      (D)  $\lim_{x \rightarrow 0} \frac{\sin 5x}{\tan 9x} = \frac{5}{9}$ .

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◎ 填空題 (五題，每題六分，共三十分，答錯不倒扣)

(1) The absolute maximum of the function  $f(x) = 2 \sin x - \cos 2x$  on the interval  $[0, 2\pi]$  is 3.

(2) Find the limit  $\lim_{\Delta x \rightarrow 0} \frac{\sin[(\pi/6) + \Delta x] - (1/2)}{\Delta x} = \underline{\frac{\sqrt{3}}{2}}$ .

(3) Find the area  $A$  of the largest rectangle that can be inscribed (內接) in a semicircle of radius  $r$ . Then  $A = \underline{r^2}$ .

(4) All points of inflection of the graph of the function  $f(x) = \sin x + \cos x$  are  $(\frac{3\pi}{4}, 0), (\frac{7\pi}{4}, 0)$ .

(5)  $\int_{-1}^1 \frac{\tan x}{1 + x^2 + x^4} dx = \underline{0}$ .

Basic Integration Rules ( $a > 0$ )

(1)  $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C;$

(2)  $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C;$

(3)  $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C.$

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系別：\_\_\_\_\_ 姓名：\_\_\_\_\_ 學號：\_\_\_\_\_

總分	初閱		複閱	
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會考成績	
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◎ 單選擇題 (單選十題，每題四分，共四十分，答錯不倒扣)

(1) C	(2) B	(3) A	(4) A	(5) D
(6) C	(7) A	(8) D	(9) B	(10) B

評分	初閱	
	複閱	

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(1) ACD	(2) BC	(3) CD
(4) ABCD	(5) BD	

評分	初閱	
	複閱	

◎ 填充題 (五題，每題六分，共三十分，答錯不倒扣)

(1) 3	(2) $\frac{\sqrt{3}}{2}$	(3) $r^2$
(4) $(\frac{3\pi}{4}, 0), (\frac{7\pi}{4}, 0)$	(5) 0	

評分	初閱	
	複閱	