

國立高雄大學理學院 108 學年度第 1 學期
微積分基礎能力會考試題 (A 卷)

◎ 單選擇題 (單選十題，每題四分，共四十分，答錯不倒扣)

(1) Which one of the following functions is even?

(A) $f_1(x) = |x^2 - 5x + 1|$.

(B) $f_2(x) = x^4 - 3x^2 + x$.

(C) $f_3(x) = \sqrt{x-1}$.

(D) $f_4(x) = \sin(x^2 + 1)$.

(2) Let $f(x) = x^3 + 2$. Which one of the following points does NOT lie on the graph of f^{-1} ?

(A) (2, 0).

(B) (4, 2).

(C) (3, 1).

(D) (1, -1).

(3) Which one of the following limits is false?

(A) $\lim_{x \rightarrow 2\pi^-} x \csc x = -\infty$.

(B) $\lim_{x \rightarrow 0^-} \frac{|x|}{x} = -1$.

(C) $\lim_{x \rightarrow 0} x^2 \sin \frac{1}{x} = 1$.

(D) $\lim_{x \rightarrow 0} \frac{\sqrt{9+x} - 3}{x} = \frac{1}{6}$.

(4) Let f be continuous on $[a, b]$, differentiable on (a, b) , and satisfy

$$\lim_{h \rightarrow 0^+} \frac{f(a+h) - f(a)}{h} < 0 \quad \text{and} \quad \lim_{h \rightarrow 0^-} \frac{f(b+h) - f(b)}{h} > 0.$$

Which one of the following statements is true?

(A) f must attain an absolute minimum at some number in (a, b) .

(B) f must attain an absolute maximum at some number in (a, b) .

(C) f must change its concavity at some number in (a, b) .

(D) none of the above.

(5) Which one of the following statements is true?

(A) If $f(x) > 1$ for all x and $\lim_{x \rightarrow 0} f(x)$ exists, then $\lim_{x \rightarrow 0} f(x) > 1$.

(B) If f is continuous at a , so is $|f|$.

(C) If $|f|$ is continuous at a , so is f .

(D) none of the above.

(6) Consider the graphs of the following functions. Which one has a vertical asymptote?

(A) $f_1(x) = \frac{x^2 - 1}{x^2 + 1}$.

(B) $f_2(x) = \frac{x^3}{x^2 + 1}$.

(C) $f_3(x) = \frac{\sqrt{2x^2 + 1}}{3x - 5}$.

(D) none of the above.

(7) Let $f(x) = \ln \frac{x+1}{\sqrt{x-2}}$. $f'(3) = ?$

(A) $\frac{1}{4}$.

(B) $-\frac{1}{4}$.

(C) $\frac{1}{2}$.

(D) $-\frac{1}{2}$.

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- (8) Given $y^2(y^2 - 4) = x^2(x^2 - 5)$, find y' at the point $(x, y) = (0, -2)$.
(A) -2 . (B) -1 . (C) 0 . (D) 1 .
- (9) Let $f(x) = \int_{\cos x}^{\sin x} \ln(1 + 2v) dv$. Then $f'(\frac{\pi}{2}) = ?$
(A) 3 . (B) $\ln 3$. (C) 1 . (D) 0 .
- (10) Evaluate the definite integral $\int_0^{\sin^{-1} \frac{3}{\sqrt{10}}} \frac{dx}{\cos^2 x \sqrt{1 + \tan x}}$.
(A) 2 . (B) 4 . (C) 3 . (D) 6 .

◎ 多選擇題 (多選五題，每題六分，共三十分。答錯一個選項扣三分，錯兩個選項以上不給分，分數不倒扣)

(1) Which of the following statements are true?

- (A) If $x_1 < x_2$ and f is a decreasing function, then $f(x_1) > f(x_2)$.
(B) If $0 < a < b$, then $\ln a < \ln b$.
(C) $\tan^{-1}(-1) = \frac{3\pi}{4}$.
(D) $\tan^{-1} x = \frac{\sin^{-1} x}{\cos^{-1} x}$.

(2) Define

$$f(x) = \begin{cases} x^3 \sin \frac{1}{x} & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

Which of the following statements are true?

- (A) f is continuous on \mathbb{R} . (B) f is differentiable on \mathbb{R} .
(C) f' is continuous on \mathbb{R} . (D) f' is differentiable on \mathbb{R} .

(3) Let $f(x) = x \sin x$. Which of the following statements are true?

- (A) $f^{(16)}(0) = 0$. (B) $f^{(16)}\left(\frac{\pi}{6}\right) = -8 + \frac{\sqrt{3}\pi}{12}$.
(C) $f^{(16)}\left(\frac{\pi}{4}\right) = -8\sqrt{2} + \frac{\sqrt{2}\pi}{8}$. (D) $f^{(16)}\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$.

(4) Which of the following statements are true?

- (A) If $3 \leq f'(x) \leq 5$ for all values of x , then $18 \leq f(8) - f(2) \leq 30$.
(B) $|\sin x| \leq |x|$ for $x \in \mathbb{R}$.
(C) $|\cos x| \leq |x|$ for $x \in \mathbb{R}$.
(D) $|\cos a - \cos b| \leq |a - b|$ for all a and b .

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(5) Which of the following statements are true?

(A) $\int_{-2}^1 \frac{1}{x^4} dx = -\frac{3}{8}$.

(B) $\int_{-1}^1 \left(x^5 - 6x^9 + \frac{\sin x}{(1+x^4)^2} \right) dx = 0$.

(C) If f is continuous on $[a, b]$, then $\frac{d}{dx} \left(\int_a^b f(x) dx \right) = f(x)$.

(D) If f' is continuous on $[1, 3]$, then $\int_1^3 f'(x) dx = f(3) - f(1)$.

◎ 填充題 (五題，每題六分，共三十分，答錯不倒扣)

(1) The domain of the function $f(x) = \sin^{-1}(3x + 1)$ is $[-2/3, 0]$.

(2) Find the values of a and b that make f continuous everywhere.

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2}, & \text{if } x < 2, \\ ax^2 - bx + 3, & \text{if } 2 \leq x < 3, \\ 2x - a + b, & \text{if } x \geq 3. \end{cases}$$

$(a, b) =$ $(1/2, 1/2)$.

(3) Use a linear approximation to estimate $\cos 29^\circ$. Then $\cos 29^\circ \approx$ $\frac{\sqrt{3}}{2} + \frac{\pi}{360}$.

(4) $\lim_{x \rightarrow 1} (2 - x)^{\tan(\pi x/2)} =$ $e^{2/\pi}$.

(5) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{3}{n} \sqrt{1 + \frac{3i}{n}} =$ $\frac{14}{3}$.

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系別：_____ 姓名：_____ 學號：_____

總分	初閱		複閱	
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會考 成績	
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◎ 單選擇題 (單選十題，每題四分，共四十分，答錯不倒扣)

(1) D	(2) B	(3) C	(4) A	(5) B
(6) C	(7) B	(8) C	(9) D	(10) A

評 分	初閱	
	複閱	

◎ 多選擇題 (多選五題，每題六分，共三十分。答錯一個選項扣三分，錯兩個選項以上不給分，分數不倒扣)

(1) AB	(2) ABC	(3) CD
(4) ABD	(5) BD	

評 分	初閱	
	複閱	

◎ 填充題 (五題，每題六分，共三十分，答錯不倒扣)

(1) $[-2/3, 0]$	(2) $(1/2, 1/2)$	(3) $\frac{\sqrt{3}}{2} + \frac{\pi}{360}$
(4) $e^{2/\pi}$	(5) $\frac{14}{3}$	

評 分	初閱	
	複閱	