一百零六學年度第二學期微積分會考試題(B 卷)

一、單選擇題 (單選十四題，每題五分，共七十分，答錯不倒扣)

1. The interval of convergence for $\sum_{n=0}^{\infty}\frac{(-3)^n x^n}{\sqrt{n+1}}$ is

(A) $[\frac{-1}{3}, \frac{1}{3}]$; (B)$[\frac{-1}{3}, \frac{1}{3})$; (C)$(\frac{-1}{3}, \frac{1}{3})$; (D)$(\frac{-1}{3}, \frac{1}{3}]$.

2. Let $f(x) = \tan^{-1}\left(\frac{x}{1-x}\right)$ Find $f'(x)$ =

(A)$\frac{1}{1-2x+2x^2}$; (B) $\frac{1}{\sqrt{1-x^2}}$; (C) $\frac{1}{x+(1-x)^2}$; (D) $\frac{(1-x)^2}{x^2+(1-x)^2}$.

3. Let $f(x) = \int_{x^2}^{0}(t^2+1)dt + \int_{0}^{x^3}(t^2-1)dt$. Find $f'(1) =$

(A) -4 (B) -3 (C) -2; (D) -1;

4. A right circular cone is generated by revolving the region bounded by $y = 0$, $x = 0$, and $y = 4 - \frac{4x}{3}$ about y-axis. Find the lateral surface area of the cone.

(A) 12π; (B) 13π; (C) 14π; (D) 15π.

5. Let $y = xtanh^{-1}(x) + \ln\sqrt{1-x^2}$. Then $\frac{dy}{dx} =$

(A) $\frac{x}{\sqrt{1-x^2}}$; (B) $\frac{1}{\sqrt{1-x^2}}$; (C) $tanh^{-1}x$; (D) $cosh^{-1}x$

6. Evaluate $\lim_{x\to 0^+} x^{\sqrt{x}}$

(A) e^{-1}; (B) 1; (C) e; (D) 0.

7. Let $x = t^3 - 6t, y = t^2$ and find an equation of the tangent line at the point where the curve crosses itself.

(A)$y = \frac{2}{3}x + \frac{8}{3}$; (B) $= \frac{-2}{3}x - \frac{7}{3}$; (C) $y = \frac{-1}{\sqrt{6}}x + 6$; (D) $y = \frac{2}{7}x + \frac{45}{7}$.

8. Evaluate $\int_0^{\pi/4}\tan^3\theta\sec^2\theta\, d\theta$

(A) 1 ; (B) $\frac{1}{2}$; (C) $\frac{1}{4}$; (D) $\frac{1}{8}$;

9. Find the arc length of the curve $x = a(\theta - sin\theta), y = a(1 - cos\theta)$ on the interval $[0, 2\pi]$

(A) $2a$; (B) $4a$; (C) $6a$; (D) $8a$;

10. The limit $\lim_{h\to 0}\frac{1}{h}\int_{-h}^{h}\cos^3 t\, dt$

(A) 3; (B) 2; (C) 1; (D) 0.

一百零六學年度第二學期微積分會考試題(B 卷)

11. Evaluate $\int_0^1 \frac{t^3}{1+t^2}dt$

(A) 1; (B) $\ln\frac{1}{\sqrt{2}}$; (C) e; (D) $\ln\sqrt{\frac{e}{2}}$.

12. Evaluate $\int_{-7}^{7}\sqrt{49-x^2}\ dx$ =

(A) $\frac{49\pi}{4}$; (B) $\frac{49\pi}{2}$; (C) 49π; (D) 0.

13. The integral $\int_0^1 \arcsin(x^2)x\ dx$ =

(A) $\frac{1}{2}(\pi-1)$; (B) $\frac{1}{4}(\pi-2)$; (C) $\frac{1}{2}(\pi+1)$; (D) $\frac{1}{4}(\pi-1)$.

14. The area of the region bounded by the curves $y=\ln(x)$, $y=0$, and $x=e$ is
(A) 1; (B) 2; (C) ln(2); (D) 2ln(2).

二、多選擇題 (多選五題，每題六分，共三十分。答錯一個選項扣三分，錯兩個選項以上不給分，分數不倒扣)

15. Which of the following statements are **Not True** ?
(A) If $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ is convergent.
(B) If $0 \le a_n \le b_n$ and $\sum b_n$ converges, then $\sum a_n$ converges.
(C) If $0 < a_n$ and $\sum a_n$ converges, then $\sum(-1)^n a_n$ converges.
(D) If $\{a_n\}$ and $\{b_n\}$ are divergent, then $\{a_n b_n\}$ is divergent

16. Which of the following statements are **True** ?
(A) The graph of the parametric equations $x=t^2, y=t^2$ is the line $y=x$.
(B) The curve represented by the parametric equation $x=t$ and $y=\cos(t)$ can be written as an equation of the form $y=f(x)$.
(C) The polar equations $r=\sin(2\theta)$, $r=-\sin(2\theta)$ and $r=\sin(-2\theta)$ all have same graph.
(D) The curve given by $x=t^3, y=t^2$ has a horizontal tangent at the origin.

17. Consider the volumes of the solids generated by revolving the region bounded by $y=0$, $x=1, x=5$ and $y=\frac{10}{x^2}$ about the given lines. Which of the following statements are **True** ?

(A) The x-axis. Then volume is $\frac{49}{15}\pi$;

(B) The y-axis. Then volume is $20\pi\ln(5)$;

(C) The line $y=-2$. Then volume is $\frac{1906}{15}\pi$;

(D) The line $y = 0$. Then volume is $\frac{496}{15}\pi$.

18. Which of the following values of x the series $\sum_{n=0}^{\infty} \frac{nx^n}{(n+1)(2x+1)^n}$ converges absolutely ?

(A) $x = -1$; (B) $x = -2$; (C) $x = 1$; (D) $x = -0.25$.

19. Which of the following statements are **Not True** ?

(A) $\arcsin^2(x) + \arccos^2(x) = 1$;

(B) $\frac{d}{dx}\left[\int_{u(x)}^{v(x)} f(t)dt\right] = f(v(x))v'(x) - f(u(x))u'(x)$

(C) If $f(x)$ is continuous on [a,b], then $f(x)$ is integrable on [a,b];

(D) $\int_{-1}^{2} \frac{1}{x}\, dx = \ln(2)$.