國立高雄大學理學院 107 學年度第 1 學期
微積分基礎能力會考試題 (B 卷)

◎ 單選擇題 (單選十題，每題四分，共四十分，答錯不倒扣)

(1) Find the values of a and b such that

$$\lim_{x\to 0}\frac{\sqrt{ax+b}-2}{x}=1.$$

(A) $(a,b)=(1,2)$. (B) $(a,b)=(4,2)$. (C) $(a,b)=(1,4)$. (D) $(a,b)=(4,4)$.

(2) Find $\dfrac{d^{2019}}{dx^{2019}}\left(2019\sin(x+108)+x^{15}\right)$.

(A) $2019\cos(x)$. (B) $\cos(x+108)$.

(C) $-2019\cos(x+108)$. (D) $2019\sin(x)+15$.

(3) Find the derivative of $\ln\left(\ln(x+2)+2\right)^2$.

(A) $\dfrac{2}{(\ln(x+2)+2)(x+2)}$. (B) $\dfrac{4}{(\ln(x+2)+2)(x+2)}$.

(C) $\dfrac{2}{(\ln(x+2)+2)^2(x+2)}$. (D) $\dfrac{4}{(\ln(x+2)+2)^2(x+2)}$.

(4) Find the slope of the tangent line of the curve $x\sin(2y)=y\sin(2x)$ at the point $(\frac{\pi}{8},\frac{\pi}{8})$.

(A) -2. (B) -1. (C) 1. (D) 2.

(5) Use a linear approximation to approximate $\sin\left(\dfrac{46}{180}\pi\right)$.

(A) $\dfrac{\sqrt{2}}{2}\left(1-\dfrac{\pi}{180}\right)$. (B) $\dfrac{\sqrt{2}}{2}\left(1+\dfrac{\pi}{180}\right)$. (C) $\dfrac{\sqrt{2}}{2}\left(1+\dfrac{\pi}{90}\right)$. (D) $\dfrac{\sqrt{2}}{2}\left(1+\dfrac{\pi}{45}\right)$.

(6) Define $f(x)=\int_{-1}^{x}xg(t)dt$, where $g(t)$ is a continuous function on $\mathbb{R}$ and it satisfies $\int_{-1}^{1}g(t)dt=2$ and $g(1)=-1$. Find $f'(1)$.

(A) -1. (B) 1. (C) -2. (D) 2.

(7) Define $f(x)=\lfloor\frac{x}{2}\rfloor-\lfloor\frac{-x}{2}\rfloor$, where $\lfloor x\rfloor$ takes the greatest integer which is smaller or equal to x. Which one of the following statements is **false**?

(A) $f(x)$ is an odd function. (B) $\lim_{x\to 1}f(x)$ exists.

(C) $\lim_{x\to 0}f(x)$ exists. (D) $\lim_{x\to -1}f(x)$ exists.

(8) Define $f(x)=\frac{e^x+e^{-x}}{2}$. Find $f^{-1}(x)$.

(A) $\ln(x+\sqrt{x^2+1})$. (B) $\ln(x-\sqrt{x^2+1})$.

(C) $\ln(x+\sqrt{x^2-1})$. (D) $\ln(x-\sqrt{x^2-1})$.

(9) Evaluate the definite integral $\int_{-\pi}^{\pi}|\cos(x)|\,dx$.

(A) 0. (B) 1. (C) 2. (D) 4.

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(10) Evaluate the definite integral $\int_{-2}^{2} \frac{x^2+2\tan(x/2)}{4+x^2}dx$.

(A) $4-\pi$. (B) $4-\frac{\pi}{2}$. (C) $4-2\pi$. (D) $2-\pi$.

◎ 多選擇題 (多選四題，每題六分，共二十四分。答錯一個選項扣三分，錯兩個選項以上不給分，分數不倒扣)

(1) Define $f(x)=(x^2+x^3)^{1/3}$, $x\in\mathbb{R}$. Which of the following statements are true?

(A) f has no horizontal asymptotes and no vertical asymptotes.

(B) f has exact two critical points.

(C) f has exact one local maximum value but no local minimum value.

(D) f has exact one inflection point.

(2) Define

$$f(x)=\begin{cases} 2\sin(x)\sin(\frac{1}{x}) & \text{if } x\neq 0,\\ 0 & \text{if } x=0.\end{cases}$$

Which of the following statements are true?

(A) The Intermediate Value Theorem can be applied to f on the interval $[-1,1]$.

(B) The Mean Value Theorem can be applied to f on the interval $[-1,1]$.

(C) f has a global maximum value.

(D) $\lim_{x\to\infty} f'(x)$ does not exist.

(3) Which of the following statements are true?

(A) $\int_0^{\pi}\tan x\sec x dx=[\sec x]_{x=0}^{\pi}=-2$.

(B) $\int_{-\pi}^{\pi}\sin(3x)\cos^2(3x)dx=0$.

(C) If f is continuous on $\mathbb{R}$, then $\int_a^b f(x+h)dx=\int_{a+h}^{b+h} f(x)dx$.

(D) If f is continuous on $\mathbb{R}$, then $\int_a^b f(x)dx=-\int_a^b f(a+b-x)dx$.

(4) Which of the following statements are true?

(A) If f is continuous at a, so is $|f|$.

(B) If f is continuous at a, then f is differentiable at a.

(C) If f is continuous on $[0,1]$ and $2\le f(x)\le 4$ on $[0,1]$, then $1<\int_0^1 f(x)dx<5$.

(D) If f is continuous on (a,b), then f has a global maximum on (a,b).

◎ 填空題 (六題，每題六分，共三十六分，答錯不倒扣)

(1) The value of $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{in^2}{n^4+i^4}$ is $\underline{\quad \pi/8 \quad}$.

(2) The value of $\lim_{x\to 0}\left(\frac{1}{\ln(x+1)}-\frac{x+1}{x}\right)^2$ is $\underline{\quad 1/4 \quad}$.

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(3) The value of $\int_0^2 \frac{x+3}{\sqrt{2-(x-1)^2}}dx$ is $\underline{\quad 2\pi \quad}$.

(4) Define $F(x) = \int_{x^3}^{8} \sqrt[3]{2^8+2^t}dt$. Then $(F^{-1})'(0) = \underline{\quad -1/96 \quad}$.

(5) Find the value of k such that two functions $f(x) = e^x$ and $g(x) = kx^2$ have exact two intersection points on $(-\infty, \infty)$. $k = \underline{\quad e^2/4 \quad}$.

(6) (See Figure 1) A man at a point A on the shore (岸邊) of a circular lake with radius 10 km wants to reach another point B on the shore. He could run along the shore of the lake to B, or swim directly to B, or first run along the shore of the lake and then swim to B. If he can run at the rate of 4 km/h and swim at the rate of 2 km/h, then the shortest time he can reach B is $\underline{\quad 5\pi/2 \quad}$.

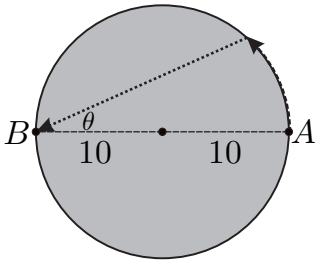

Figure 1

Basic Integration Rules ($a > 0$)

(1) $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C;$

(2) $\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C;$

(3) $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\text{arcsec}\frac{|u|}{a} + C.$

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系別：＿＿＿＿＿＿＿＿　　姓名：＿＿＿＿＿＿＿＿　　學號：＿＿＿＿＿＿＿＿

總分	初閱		複閱	

會考成績	

◎ 單選擇題 (單選十題，每題四分，共四十分，答錯不倒扣)

(1) D	(2) C	(3) A	(4) C	(5) B
(6) B	(7) C	(8) C	(9) D	(10) A

評分	初閱	
	複閱	

◎ 多選擇題 (多選四題，每題六分，共二十四分。答錯一個選項扣三分，錯兩個選項以上不給分，分數不倒扣)

(1) AD	(2) AC	(3) BC
(4) AC		

評分	初閱	
	複閱	

◎ 填空題 (六題，每題六分，共三十六分，答錯不倒扣)

(1) $\pi/8$	(2) $1/4$	(3) 2π
(4) $-1/96$	(5) $e^2/4$	(6) $5\pi/2$

評分	初閱	
	複閱	