

國立高雄大學一〇四學年度第一學期
理學院微積分基礎能力會考試題

日期：104 年 12 月 9 日 時間：120 分鐘 學號： 姓名：

Notation:

- \mathbb{Z} : the set of integers
- \mathbb{Q} : the set of rational numbers
- \mathbb{R} : the set of real numbers
- \in : belong; for example, $a \in \mathbb{R}$ means that “ a is a real number.”

I Fundamentality (30%)

1. What is the limit of $f(x) = 4$ as x approaches π ?
A. $\lim_{x \rightarrow \pi} 4 = \pi$ B. $\lim_{x \rightarrow \pi} 4 = 4$ C. $\lim_{x \rightarrow \pi} 4 = \frac{\pi}{4}$ D. $\lim_{x \rightarrow \pi} 4 = 4\pi$ E. none of the above
2. Find all the x -values at which $f(x) = \frac{\cos x^2}{x^2 - 2x}$ is not continuous.
A. $x = 0$ B. $x = 2$ C. $x = 0, 2$ D. $f(x)$ is continuous on \mathbb{R} E. none of the above
3. Find the vertical asymptotes (if any) of the function $f(x) = \tan(15x)$.
A. $x = \frac{k}{15}\pi, k \in \mathbb{Z}$ B. $x = \frac{2k}{15}\pi, k \in \mathbb{Z}$ C. $x = \frac{2k+1}{15}\pi, k \in \mathbb{Z}$ D. $x = \frac{2k+1}{30}\pi, k \in \mathbb{Z}$
E. none of the above
4. Find the derivative of $f(x) = \frac{1}{x^4}$.
A. $-\frac{4}{x^5}$ B. $\frac{4}{x^5}$ C. $-\frac{4}{x^3}$ D. $\frac{4}{x^3}$ E. none of the above
5. Find the derivative of $f(x) = \cos(2x^4 - 6)$.
A. $8x^3 \cos(2x^4 - 6)$ B. $-8x^3 \cos(2x^4 - 6)$ C. $-8 \sin(2x^4 - 6)$ D. $-8x^3 \sin(2x^4 - 6)$
E. $8x^3 \sin(2x^4 - 6)$
6. A business has a cost of $C = 1.7x + 700$ for producing x units. The average cost per unit is $\bar{C} = \frac{C}{x}$. Find the limit of \bar{C} as x approaches infinity.
A. 1 B. 700 C. 0.7 D. does not exist E. none of the above
7. Find the differential dy of $y = x \cos(7x)$.
A. $(\cos(7x) - \sin(7x))dx$ B. $(\cos(7x) + \sin(7x))dx$ C. $(\cos(7x) - 7x \sin(7x))dx$ D. $(\cos(7x) + 7x \sin(7x))dx$ E. $\cos(7x) - 7x \sin(7x)$
8. Write the limit $\lim_{|\Delta| \rightarrow 0} \sum_{i=1}^n \frac{6}{c_i^2} \Delta x_i$ as a definite integral on the interval $[8, 10]$, where c_i is any point in the i^{th} subinterval.
A. $\int_{10}^8 \frac{6}{x^2} dx$ B. $\int_8^{10} \frac{6}{x^2} dx$ C. $\int_0^{10} \frac{6}{x^2} dx$ D. $\int_0^2 \frac{6}{x^2} dx$ E. $\int_0^8 \frac{6}{x^2} dx$
9. Find the average value of $f(x) = \frac{5(x^2 + 5)}{x^2}$ on the interval $[1, 3]$.
A. 25 B. 40 C. $\frac{25}{3}$ D. $\frac{80}{3}$ E. $\frac{40}{3}$
10. Find the indefinite integral $\int 7x^6 \sin x^7 dx$.
A. $-\frac{\cos x^8}{8} + C$ B. $-\frac{\cos x^7}{7} + C$ C. $\sin x^7 + C$ D. $-\cos x^8 + C$ E. $-\cos x^7 + C$

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II Medium (40%)

11. Find the limit $\lim_{x \rightarrow -6^+} (-3\lceil x \rceil - 8)$, where $f(x) = \lceil x \rceil$ represents the greatest integer which is smaller than or equal to x .
A. **-23** B. 23 C. 10 D. -10 E. does not exist
12. Find all values of c such that $f(x) = \begin{cases} 4 - x^2, & x \leq c; \\ x, & x > c; \end{cases}$ is continuous on \mathbb{R} .
A. $\frac{-1 + \sqrt{17}}{2}$ B. 0 C. $\frac{-1 \pm \sqrt{17}}{2}$ **D. $\frac{-1 \pm \sqrt{17}}{2}$** E. none of the above
13. Find the open intervals where the function $f(x) = x\sqrt{30 - x^2}$ is increasing.
A. $(\sqrt{15}, \infty)$ B. $(-\infty, -\sqrt{30})$ C. $(\sqrt{30}, \infty)$ D. $(-\sqrt{30}, -\sqrt{15})$ and $(\sqrt{15}, \sqrt{30})$ **E. $(-\sqrt{15}, \sqrt{15})$**
14. Find the derivative of $f(x) = \int_{x^2}^4 5\sqrt{1+t^2} dt$.
A. $10x\sqrt{1+x^4}$ **B. $-10x\sqrt{1+x^4}$** C. $5x^2\sqrt{1+x^2}$ D. $-5x^2\sqrt{1+x^2}$ E. $5x\sqrt{1+x^2}$
15. Find a , such that the function $f(x) = \cos \pi x + \sqrt{a - x^2}$ has the domain $(-5, 5)$.
A. $a = 5$ B. $a = -5$ **C. $a = 25$** D. $a = -25$ E. none of the above
16. Use the linear approximation of the function $f(x) = \sqrt{9 - x}$ to approximate $\sqrt{9.09}$.
A. 0.15 B. 2.92 C. 3.12 **D. 3.02** E. none of the above
17. Find the tangent line to the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$ at the point $(2, -\sqrt{3})$.
A. $y = 1.29x - 2.31$ B. $y = -0.71x - 1.31$ C. $y = 0.39x - 3.31$ **D. $y = 0.288x - 2.308$**
E. none of the above
18. Evaluate the definite integral $\int_{-\pi/2}^{\pi/2} \frac{x^2 \tan x}{2 + x^4} dx$.
A. 0 B. 1 C. -2 D. 3 **E. none of the above**
19. Given that the graph of f passes through the point $(4, 69)$ and that the slope of its tangent line at $(x, f(x))$ is $10x - 4$. Find $f(1)$.
A. 1 B. 12 C. 11 **D. 6** E. 0
20. Suppose that f and g are differentiable functions with $f(5) = 5$ and $\lim_{x \rightarrow 5} (2f(x) - g(x)) = 6$. Find $g(5)$.
A. $g(5) = 5$ B. $g(5) = 16$ C. $g(5) = 2$ **D. $g(5) = 4$** E. $g(5) = 6$

III Challenge (30%)

21. For what values of x is $f(x) = \begin{cases} 0, & x \text{ is rational;} \\ x, & x \text{ is irrational;} \end{cases}$ continuous?
A. all rational x B. all real x C. all irrational x **D. $x = 0$** E. none of the above
22. How many real roots does the equation $x^5 - 7x + c = 0$ have in the interval $[-1, 1]$?
A. at most two real roots B. no real roots C. at most three real roots **D. at most one real root** E. none of the above

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23. Evaluate the integral $\int_{-1}^3 |2x - x^2| dx$.
A. 4.00 B. 19.33 C. 22.00 D. 14.67 E. 32.67
24. The mass of part of a wire is $x(1 + \sqrt{x})$ kilograms, where x is measured in meters from one end of the wire. Find the linear density of the wire when $x = 16$ m.
A. 5 kg/m B. 6 kg/m C. 7 kg/m D. 1.5 kg/m E. none of the above
25. Suppose that $F(x) = \int_1^x f(t) dt$ with $f(t) = \int_1^{t^2} \frac{\sqrt{2+u^2}}{u} du$. Find $F''(2)$.
A. $\frac{3\sqrt{2}}{2}$ B. $6\sqrt{3}$ C. $6\sqrt{2}$ D. $3\sqrt{3}$ E. $3\sqrt{2}$
26. Find the point on the line $y = 4x + 8$ that is closest to the origin.
A. $\left(-\frac{32}{17}, \frac{10}{17}\right)$ B. $\left(-\frac{34}{17}, \frac{9}{17}\right)$ C. $\left(-\frac{31}{17}, \frac{8}{17}\right)$ D. $\left(-\frac{32}{17}, \frac{8}{17}\right)$ E. $\left(-2, \frac{8}{17}\right)$