

NUK Math 徵答009 解答

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January 14, 2013

問題009

Show that every closed set in $\mathbb{R}$ is a countable intersection of open sets.

陳柏凱的解法

Since the statement "Every closed set in $\mathbb{R}$ is a countable intersection of open sets" is equivalent to "Every open set in $\mathbb{R}$ is a countable union of closed sets.", so we claim that the latter. First, $\mathbb{R}$ and $\emptyset$ are open and closed, so we have done it. Next, since every nonempty open subset in $\mathbb{R}$ could be written as a union of disjoint open intervals, moreover, every open interval is a class of (a, b) , $(-\infty, b)$ and (a, ∞) , $a, b \in \mathbb{R}$. In particular,

$$(a, b) = \bigcup_{n=1}^{\infty} [a + 1/n, b - 1/n], \quad (-\infty, b) = \bigcup_{n=1}^{\infty} (-\infty, b - 1/n]$$

and

$$(a, \infty) = \bigcup_{n=1}^{\infty} [a + 1/n, \infty).$$

Therefore, every open subset G in $\mathbb{R}$, there exists a countable collection F of disjoint closed intervals such that $G = \bigcup F$, then we complete the proof.

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