

高雄大學應用數學系
每月挑戰 答案紙

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Note that for any $k = 2m, m \in N$, we have

$$\left(1 + \frac{1}{k-1}\right)\left(1 - \frac{1}{k}\right) = \left(1 + \frac{1}{2m-1}\right)\left(1 - \frac{1}{2m}\right) = \left(\frac{2m}{2m-1}\right)\left(\frac{2m-1}{2m}\right) = 1.$$

Let even natural number $n = 2t$, we have

$$\begin{aligned} &\left(1 - \frac{1}{2}\right)\left(1 + \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) \dots \left(1 - \frac{(-1)^n}{n}\right) = \left(1 - \frac{1}{2}\right)\left(1 + \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) \dots \left(1 + \frac{1}{2t-1}\right)\left(1 - \frac{1}{2t}\right) \\ &= \frac{1}{2} \cdot \prod_{m=2}^t \left(1 + \frac{1}{2m-1}\right)\left(1 - \frac{1}{2m}\right) = \frac{1}{2}. \end{aligned}$$