

016 Solution

Let $a_1 < a_2 < \cdots < a_r$ be a choice of r numbers, no two consecutive, out of $\{1, 2, \cdots, n\}$. Then $a_1, a_2 - 1, a_3 - 2, \cdots, a_r - r + 1$ is a choice of r distinct numbers out of $\{1, 2, \cdots, n - r + 1\}$.

Conversely, if $b_1 < b_2 < \cdots < b_r$ is a choice of r distinct numbers out of $\{1, 2, \cdots, n - r + 1\}$. Then $b_1, b_2 + 1, b_3 + 2, \cdots, b_r + r - 1$ is a choice of r numbers, no two consecutive, out of $\{1, 2, \cdots, n\}$.

It follows from this 1-1 correspondence that there are $\binom{n-r+1}{r}$ ways to choose $a_1 < a_2 < \cdots < a_r$, no two consecutive, out of $\{1, 2, \cdots, n\}$.

Similarly, there are $\binom{n-r-1}{r-2}$ ways to choose $1 = a_1 < a_2 < \cdots < a_r = n$, no two consecutive, out of $\{1, 2, \cdots, n\}$.

Therefore, there are

$$\binom{n-r+1}{r} - \binom{n-r-1}{r-2} = \frac{n}{n-r} \binom{n-r}{r}$$

ways.