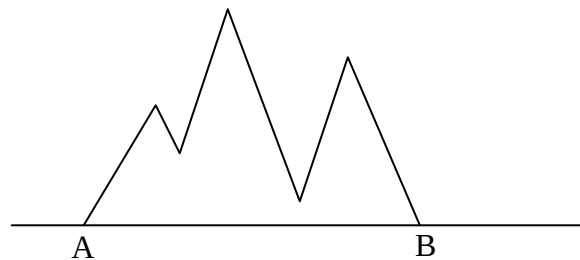


# NUK Math 徵答 023 解答：.

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023

A mountain range is a polygonal curve from  $(a, 0)$  to  $(b, 0)$  (see figure 1 for an example) in the upper half-plane. Hikers A and B begin at  $(a, 0)$  and  $(b, 0)$ , respectively. Prove that A and B can meet by traveling on the mountain range in such a way that **at all times their heights above**



the horizontal axis are the same.

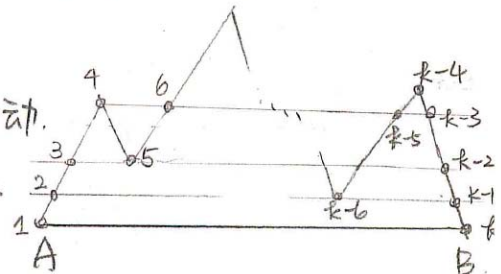
解：蔡一慈和蔡宜霖的解法

對任意的 mountain range, 在所有轉折處 (山頂 or 山谷)  
畫上水平線後, 將所有水平線與 mountain 的交點由左至右  
依序編號, (如圖所示)

此時我們定義一 graph 對應至 A, B 的移动.

令  $G = (V, E)$ , 因 A, B 必須維持同一水平線

令  $0, k$  分別為 A, B 的起點,



$$V(G) = \{v = (a, b) \mid a, b \text{ 為在同一水平上任意兩點}, a \leq b\}$$

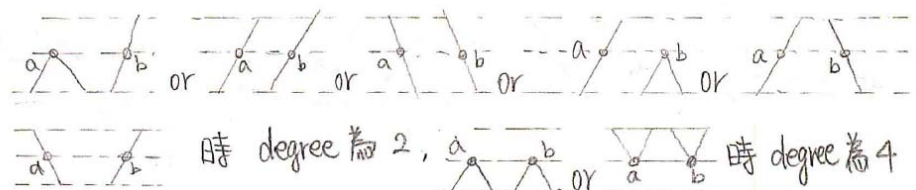
$$E(G) = \{e = \{(a, b), (c, d)\} \mid (a, b) \in V, (c, d) \in V \text{ and } |a - c| = 1, |b - d| = 1\}$$

下列分別討論  $v \in V$  的 degree:

$$(i) \deg_G((0, k)) = 1 \quad (\because (0, k) \text{ 只連到 } (1, k-1))$$

$$(ii) \deg_G(v) = 2 \text{ or } 4, \text{ where } v = (a, b), a \neq b \text{ and } (a, b) \neq (0, k)$$

即原圖為



假設  $G'$  是包含起點  $(0, k)$  的 component.

若  $A, B$  不可能相遇, 則  $G'$  不包含  $(a, a)$  for any  $(a, a) \in V$

$\Rightarrow \sum_{v \in V(G')} \deg_{G'}(v) \stackrel{\text{is}}{=} \text{odd}$  (由 (i) 和 (ii) 貢獻) (~~contradict by~~  
Degree sum formula)



答題優良名單：

蔡一慈和蔡宜霖