高雄大學應用數學系

每月挑戰 013-015

2013 年一月

1. 每個月將公布若干個數學問題徵答，由系上老師出題，歡迎同學來挑戰。
2. 同學解出任何一題皆可將解答投稿到系辦 (答案紙格式可於系網頁上下載)。系上將擇優公布優良解答，並公開表揚優秀解題同學。

013

Prove that for each even natural number n,

$$\left(1-\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\cdots\left(1-\frac{(-1)^n}{n}\right)=\frac{1}{2}.$$

[劉欽岳教授出題]

014

(a) Let $A \in M_{m\times n}(\mathbb{R}), B \in M_{n\times m}(\mathbb{R})$. Show that if $\lambda \neq 0$ is an eigenvalue of AB then λ is an eigenvalue of BA.

(b) Let $A = 3I_n - \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}[1, \cdots, 1]$. Find the eigenvalues and eigenvectors of A.

[郭岳承教授出題]

015

A set $D \subseteq \mathbb{R}^n$ is disconnected if and only if $D = E \cup F$, where E, F are nonempty and $E \cap \overline{F} = \varnothing$, $\overline{E} \cap F = \varnothing$ ($\overline{B}$ is denoted the closure of set B).

[吳宗芳教授出題]